1. In a binary tree:
   a. What is the maximum number of nodes if the depth of the tree is \( d \)?
   b. What is the internal path length of the tree that has maximum number of nodes and that has a depth of \( d \)?
   c. What is the minimum number of nodes if the depth of the tree is \( d \)?

2. The following items are inserted into a binary tree in the following order:
   1, 3, 5, 8, 0, 9, 12, 4, 2, 15
   a. Assume the binary tree is just a binary search tree (BST). Draw the tree after all items are inserted.
   b. For the BST that you obtained in a), draw the tree again after you have deleted items 5, 3, 9, 15, and 0 (deletion is in this order).
   c. Assume the binary tree is an AVL tree. Draw the tree after every insert operation and rotation (if required). Write down what kind of rotation it is, if rotation is necessary.

3. Exercise 4.8 (in textbook) – related with infix, prefix, and postfix notations.

4. Exercise 4.18 (in textbook) – related to minimum number of nodes in an AVL tree.


6. Programming exercise. Provide your answer together with program output.
   a. Implement an algorithm that takes a string (as input from user) that contains a mathematical expression on postfix notation and converts it to an expression tree. You may assume that the mathematical postfix expression that is input to your program will consists of only from addition, subtraction, multiplication and division operators, and all operands are positive integers.
   b. Traverse the tree you obtained in a) and print out the expression it represents using preorder, postorder, and inorder traversal strategies.
   c. Evaluate the expression and print out the result.

7. Programming exercise. Implement the AVL tree class and insert method (You may benefit from the AVL tree and binary search tree class definitions and methods shown in the book). Then randomly insert 100, 1000, and 10 thousand items (in 3 different runs of your program – meaning that you insert 100, 1000, and 10 thousand items into different empty trees). Items should be randomly chosen from range of integers between 1 and 10 thousand. Handle duplicates by using and increasing a frequency counter kept at every node. For every run of your program, compute and output the following:
   a. Number of single rotations, number of single left rotations, number of single right rotations.
   b. Number of double rotations, number of double left-right rotations, number of double right-left rotations.
   c. The elapsed time for running the program.
   d. The average time required to do a single rotation.
   e. The average time required to do a double rotation.