Routing Topology Algorithms

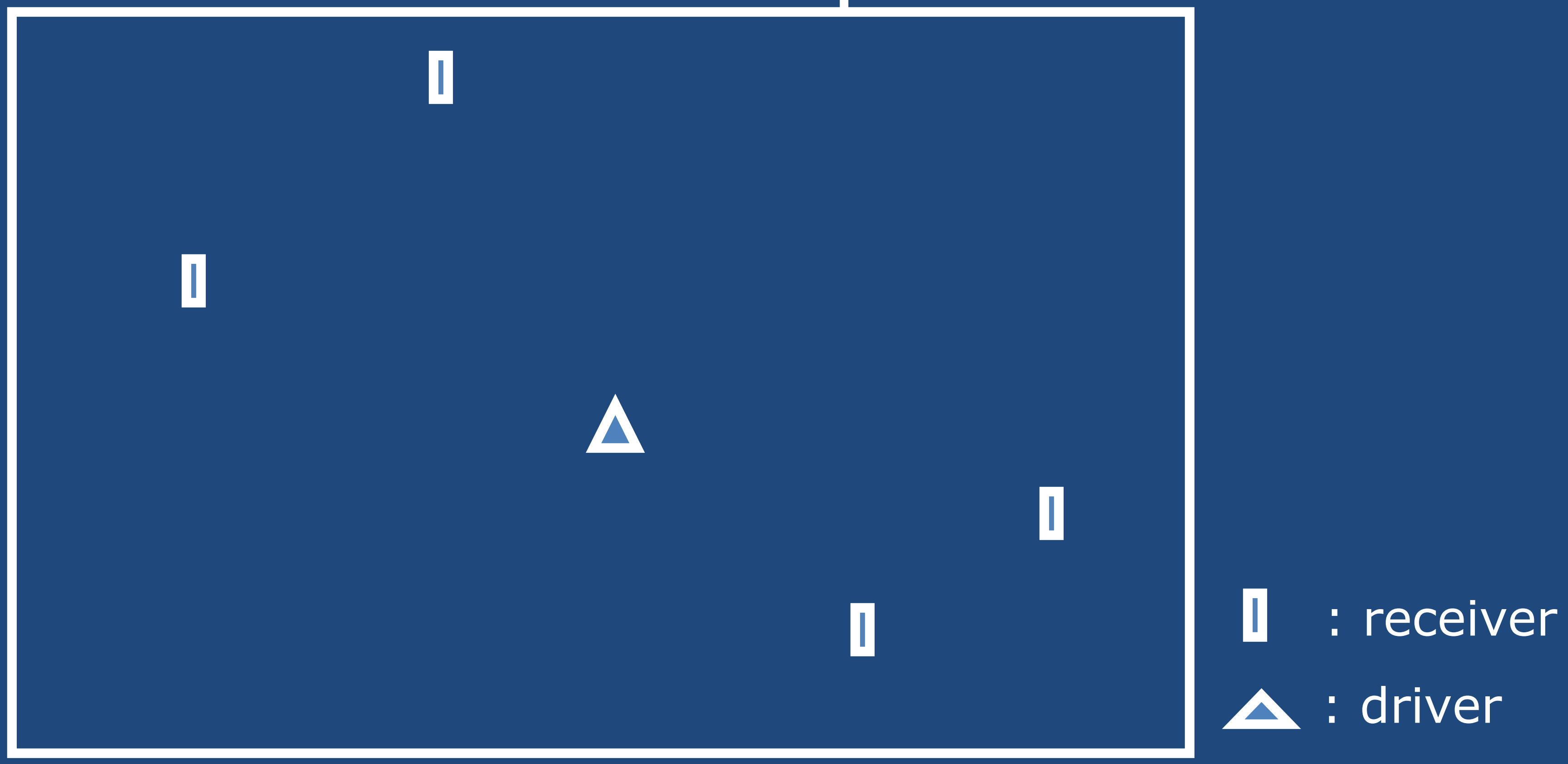
Mustafa Ozdal

Introduction

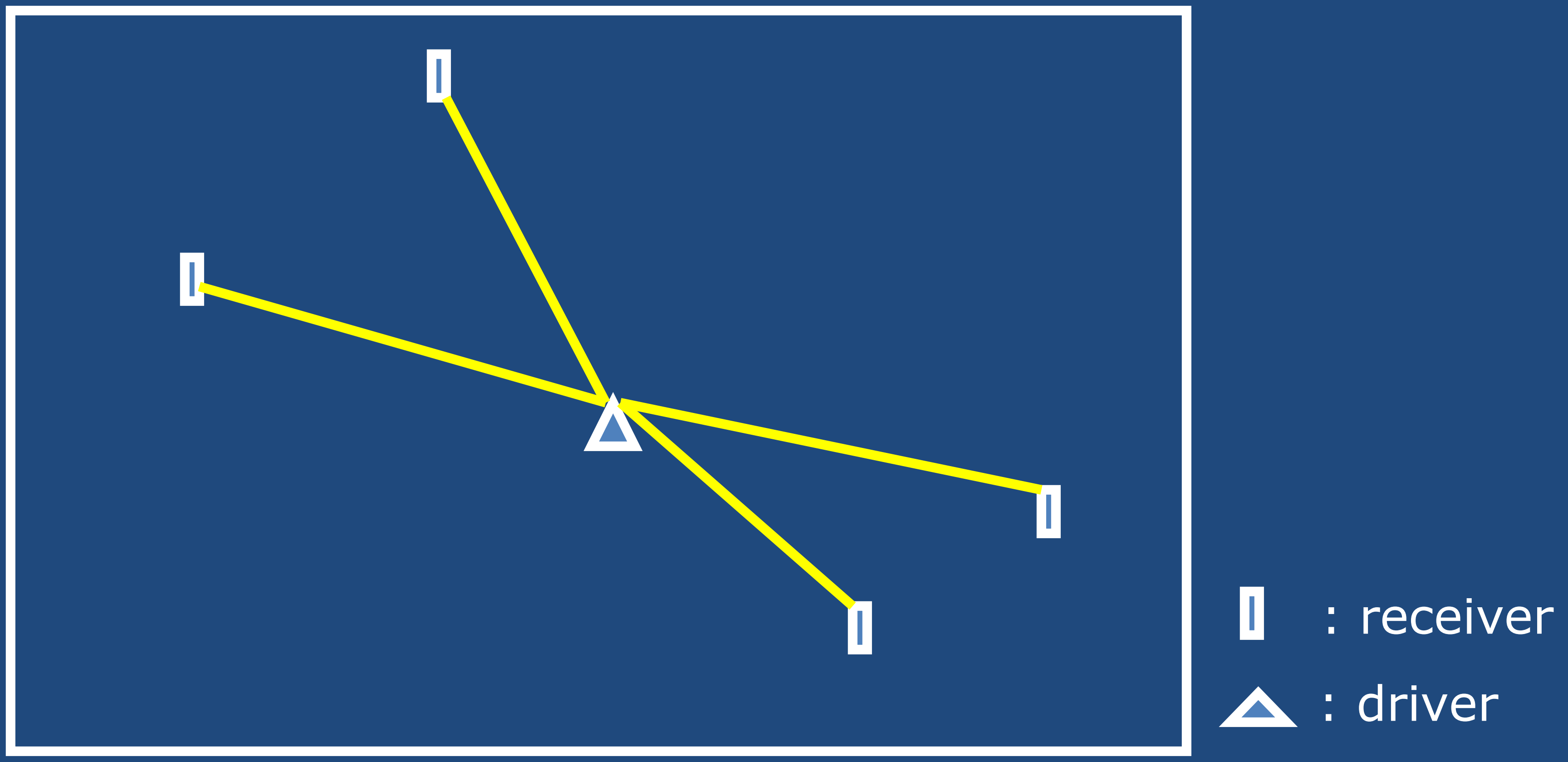
- How to connect nets with multiple terminals?
- Net topologies needed before point-to-point routing between terminals.

- Several objectives:
  - Minimum wirelength
  - Best timing
  - Routability

Example

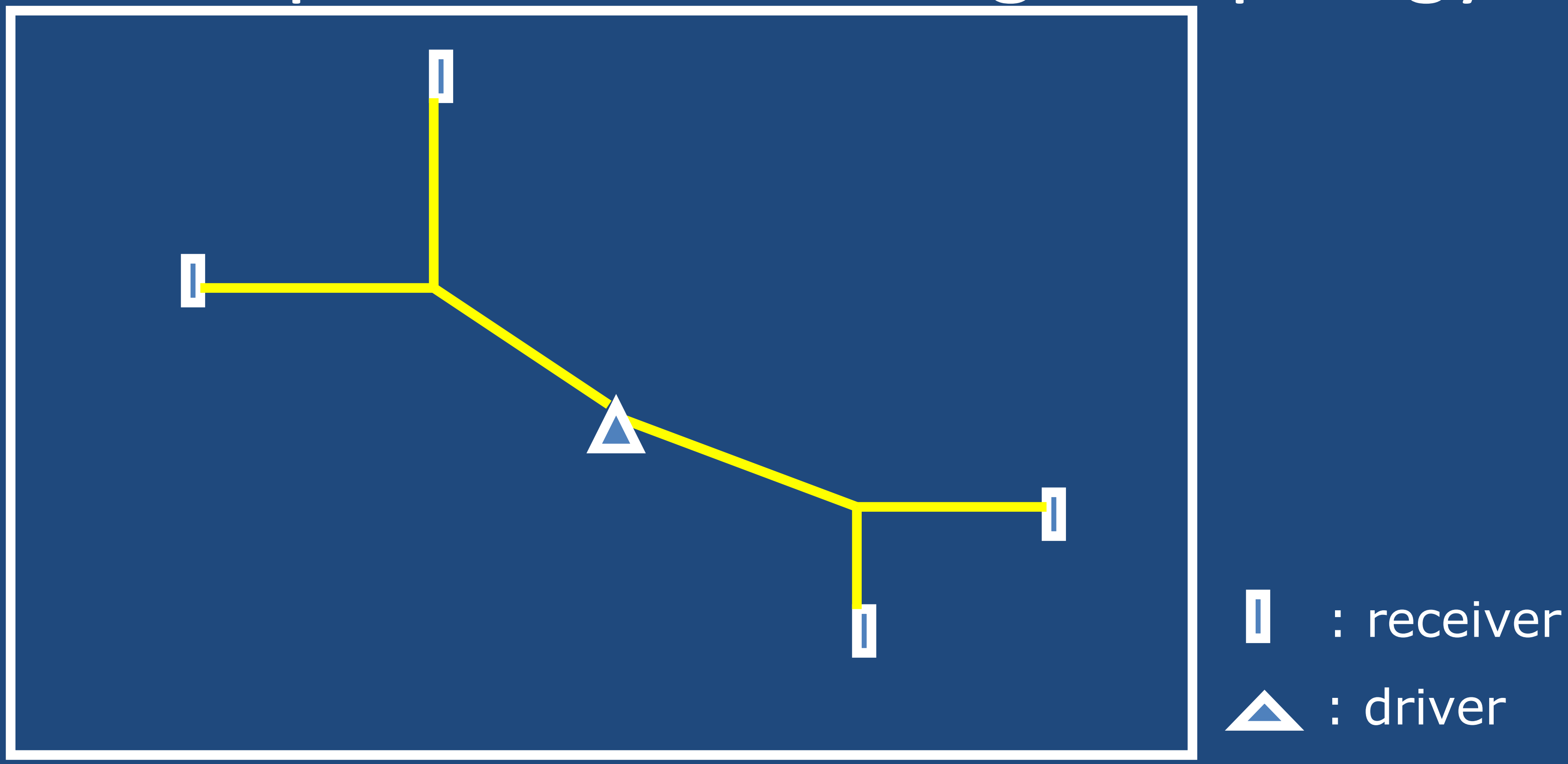


Example – Star topology (suboptimal)



Connect each receiver to the driver independently.

Example – Min Wirelength Topology



Outline

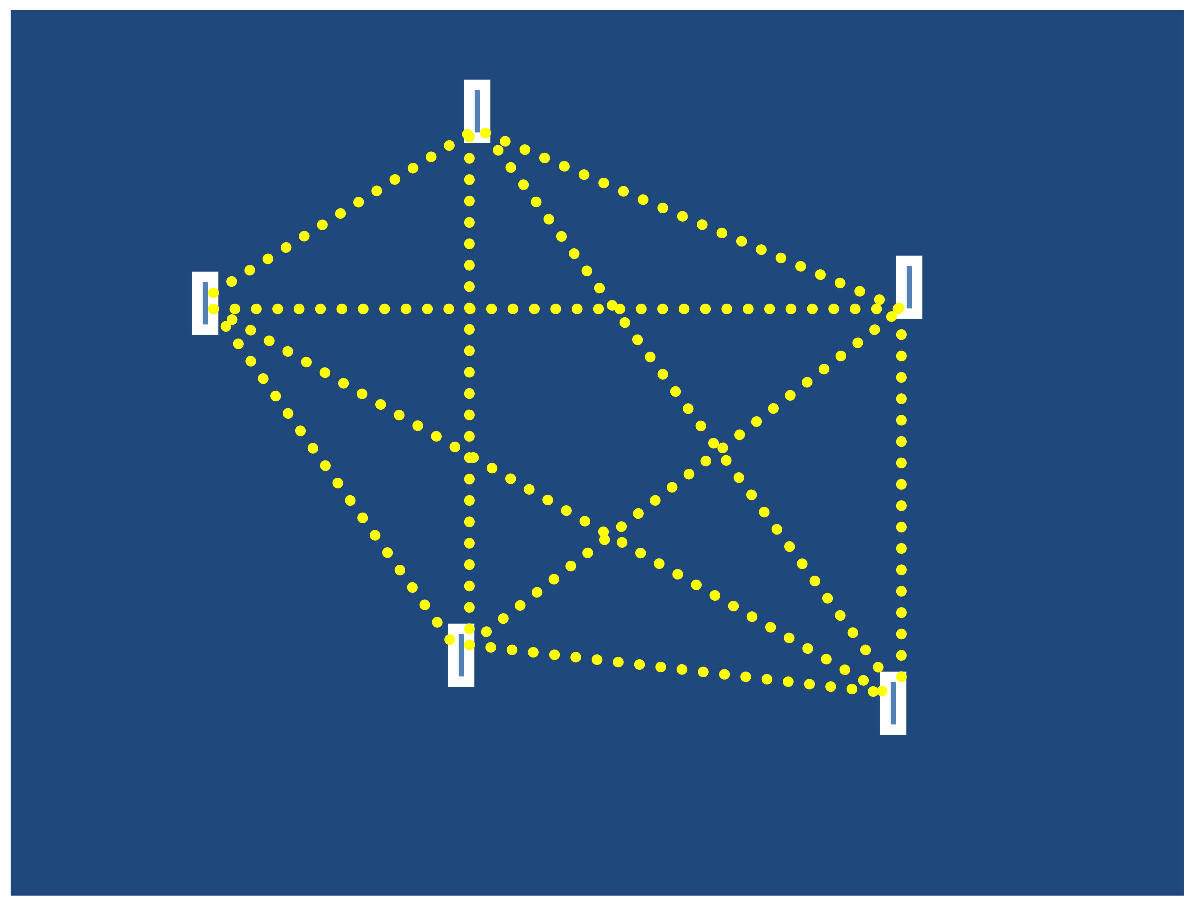
- Definitions and basic algorithms
  - Minimum Spanning Trees (MST)
  - Steiner Trees
  - Rectilinear Steiner Trees
- Wirelength vs timing tradeoff

Minimum Spanning Tree (MST)

- Consider a connected graph G = (V, E)
  - V: terminals
  - E: potential connections between terminals
  - w(e): wirelength of edge e

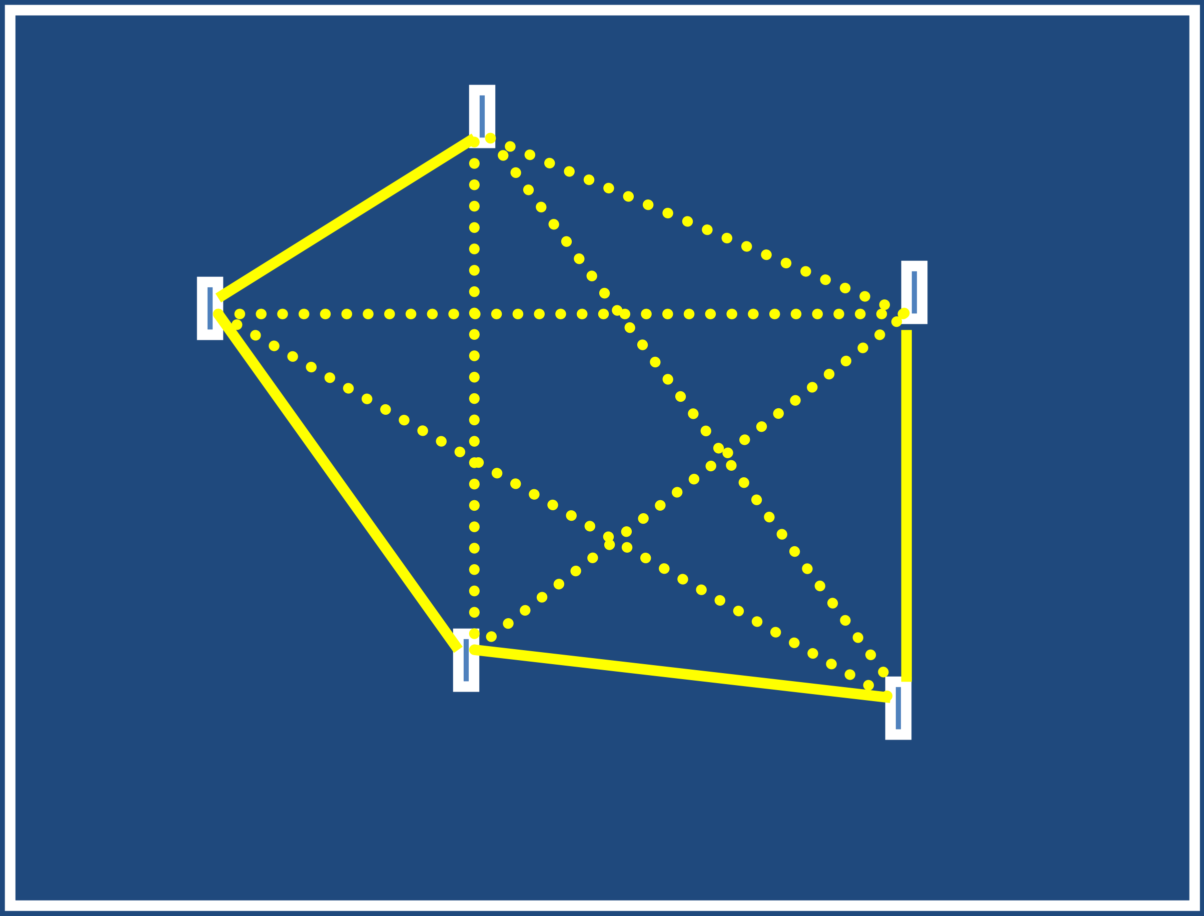
- MST: The set of edges E_T such that:
  - E_T is a subset of E
  - The graph T = (V, E_T) is connected
  - The total edge weight of E_T is minimum

MST Example



- 5 vertices
- 10 edges
- Weight of edge e is the Manhattan distance of e
- What is the MST?

MST Example



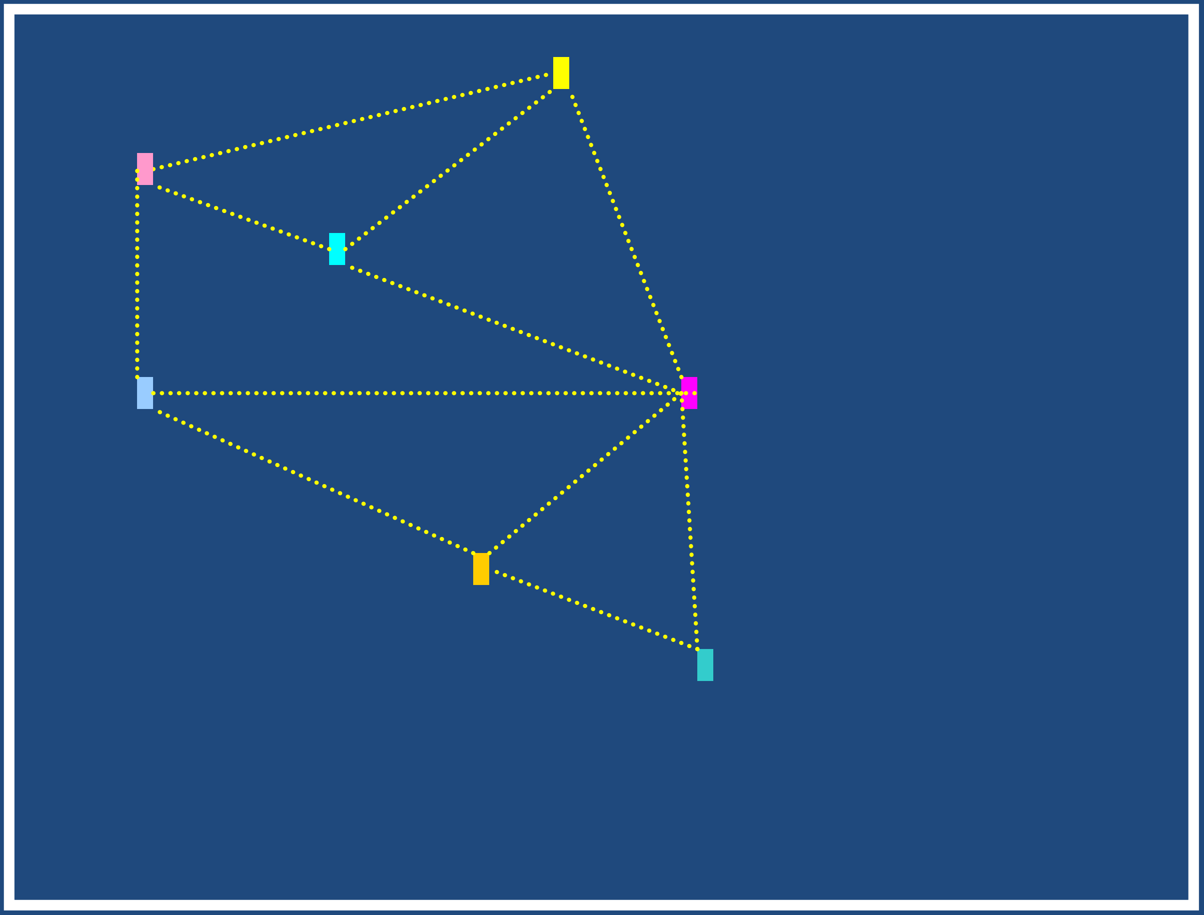
- The edge set E_T:
  - $W(E_T)$ is minimum
  - $T = (V, E_T)$ is still connected

- Note: $T = (V, E_T)$ must contain n vertices and n-1 edges.

Kruskal's MST Algorithm

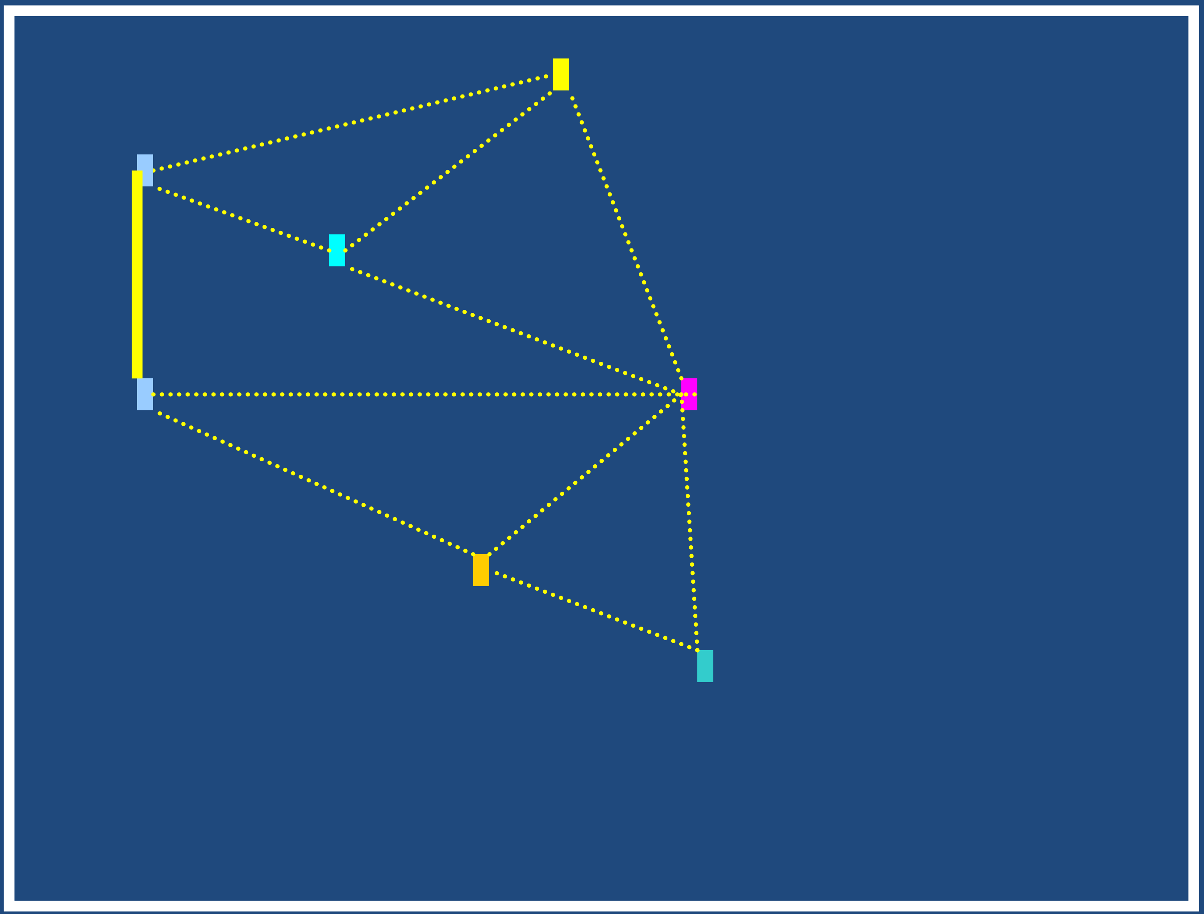
1. Initialize T as empty set
2. Define a disjoint set corresponding to each vertex in V.
3. Sort edges in non-decreasing order of weights
4. For each e = (v1, v2) in the sorted edge list
    1. If v1 and v2 belong to different sets
        1. Add e to T
        2. Merge the sets corresponding to v1 and v2
5. Return T

Kruskal’s MST Algorithm - Example



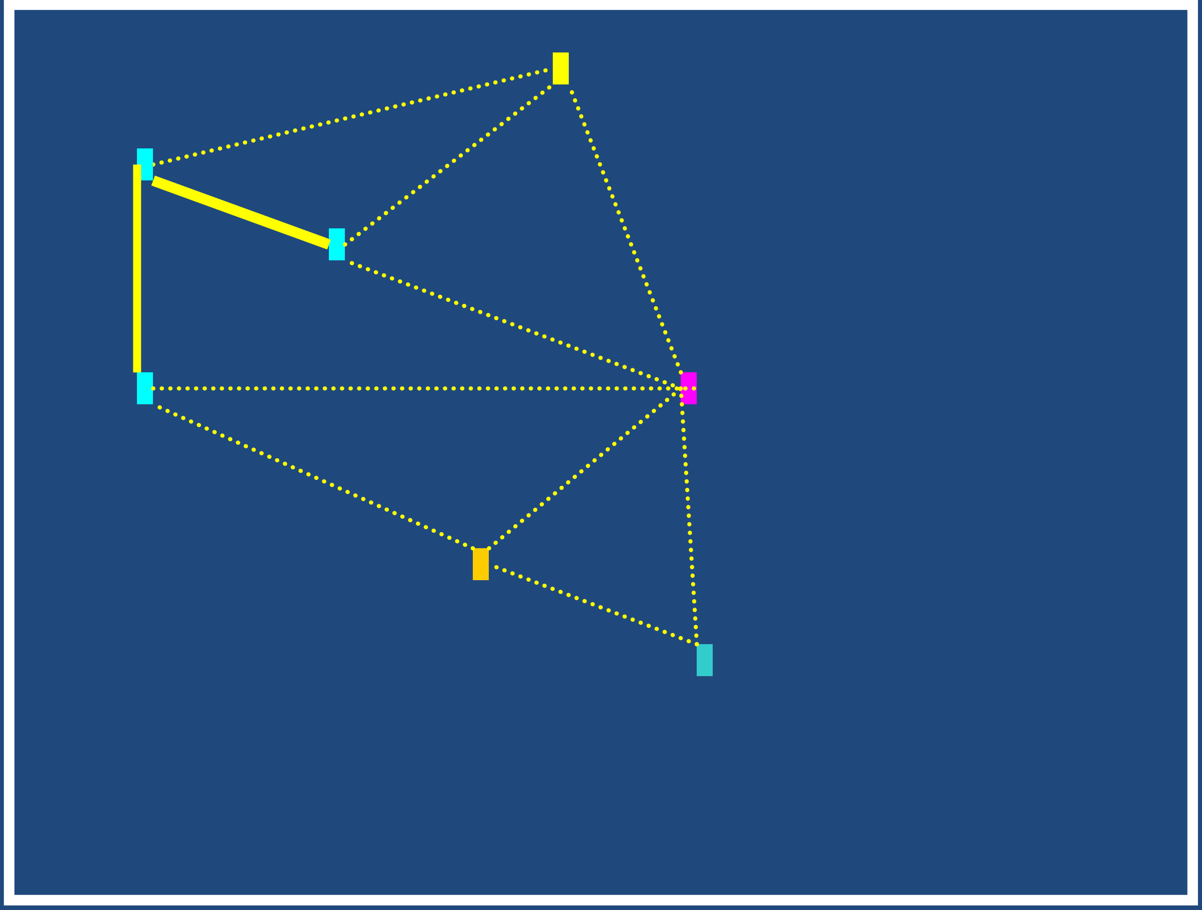
- Initially, each vertex is a disjoint set (different color)
- We will process edges from shortest to longest

Kruskal's MST Algorithm – Example
Step 1



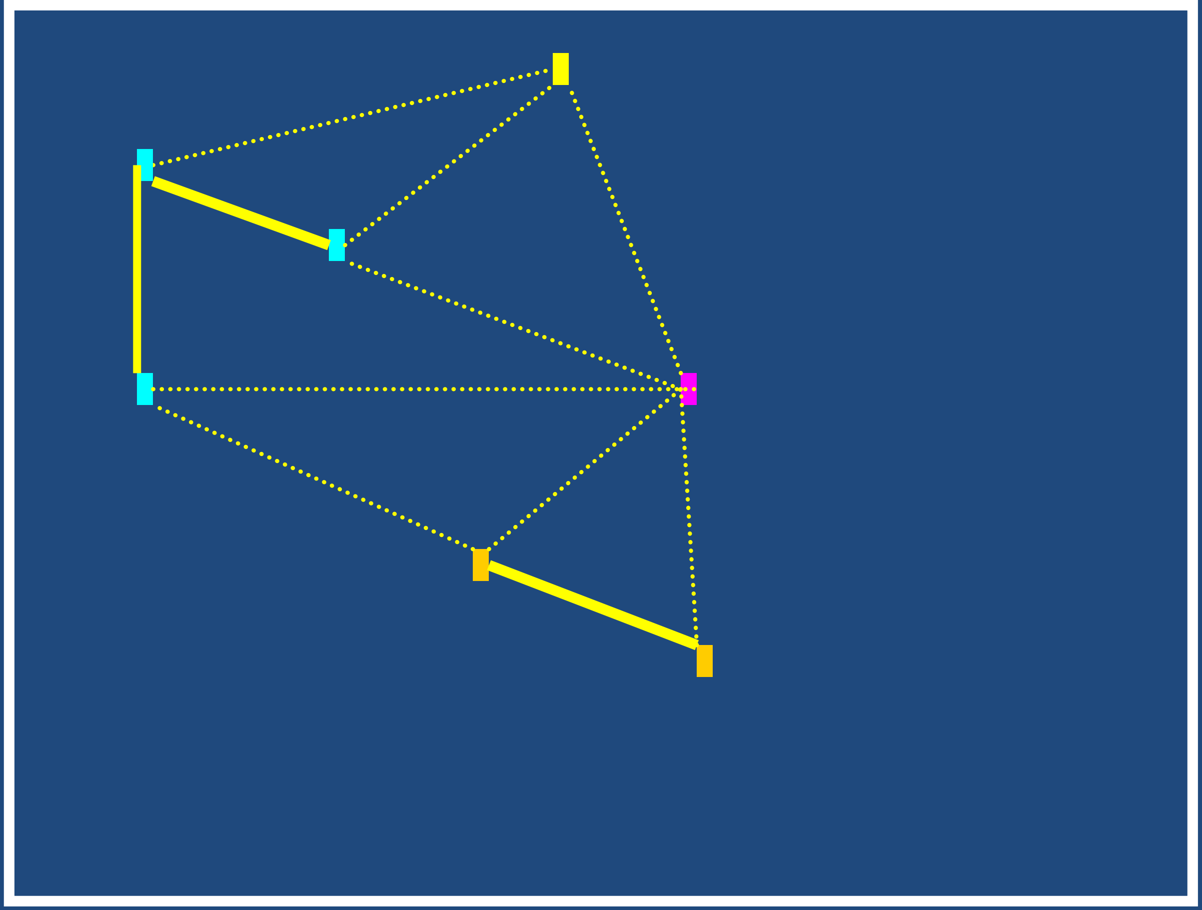
- Start from the shortest edge
- The vertices connected are in different sets
- Add the edge to MST
- Merge the vertices

Kruskal's MST Algorithm - Example Step 2



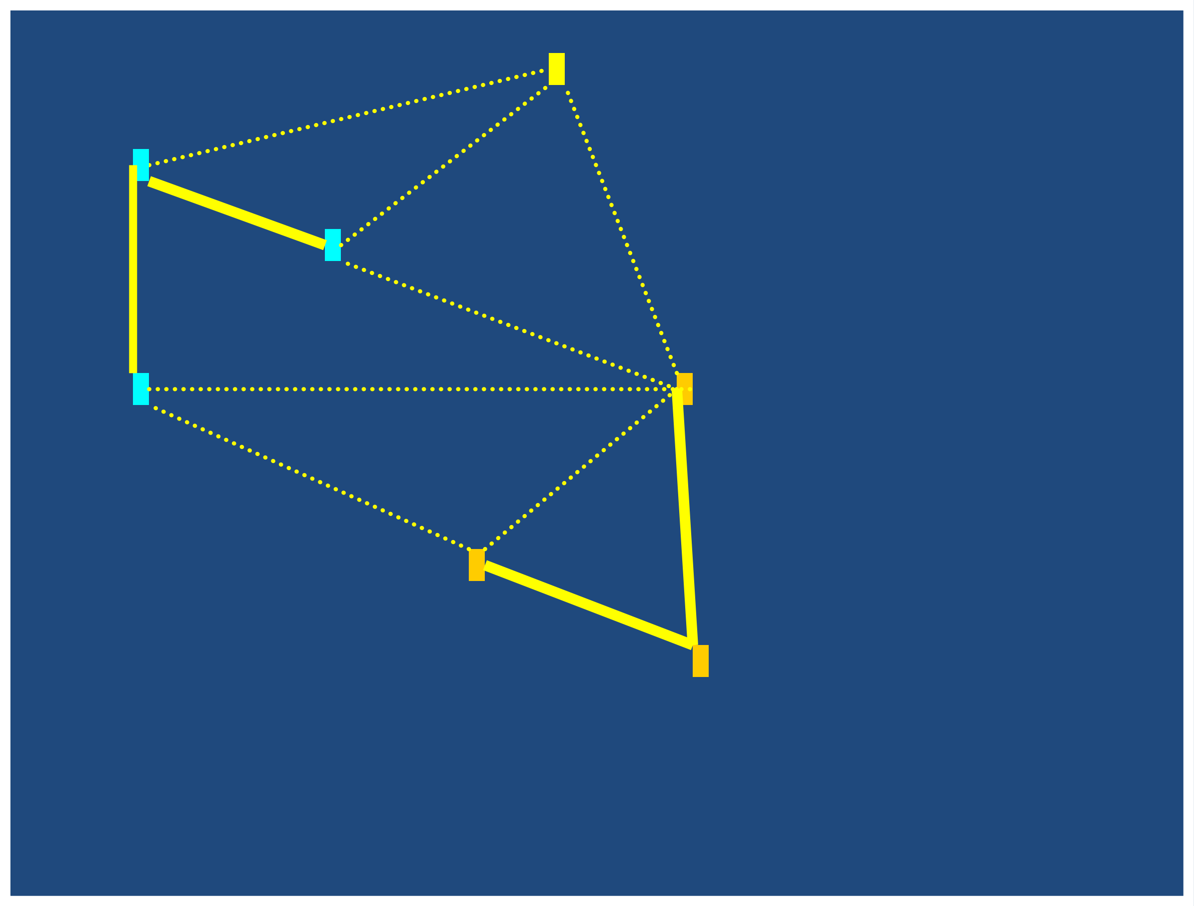
- Process the next shortest edge.
- The vertices connected are in different sets
- Add the edge to MST
- Merge the vertices

Kruskal’s MST Algorithm – Example Step 3



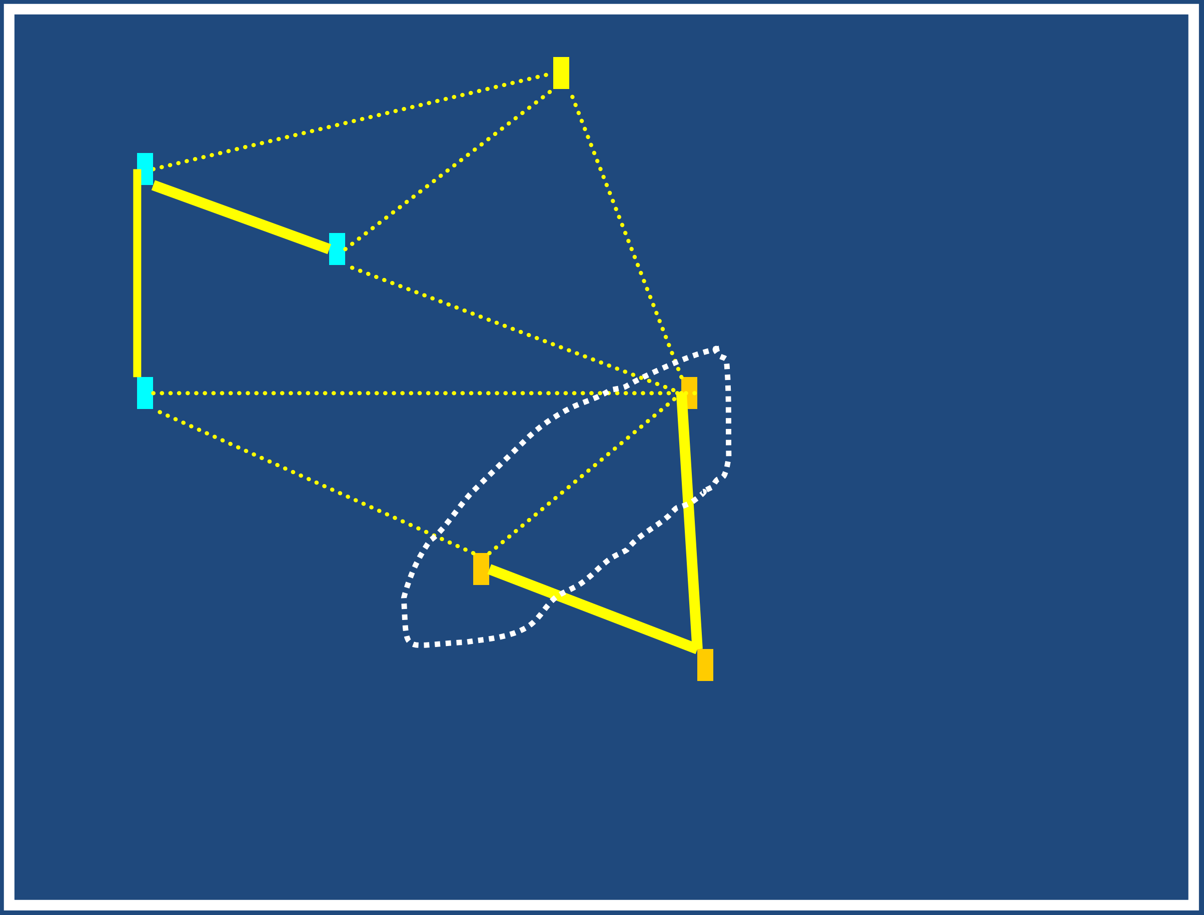
- Process the next shortest edge.
- The vertices connected are in different sets
- Add the edge to MST
- Merge the vertices

Kruskal's MST Algorithm – Example Step 4



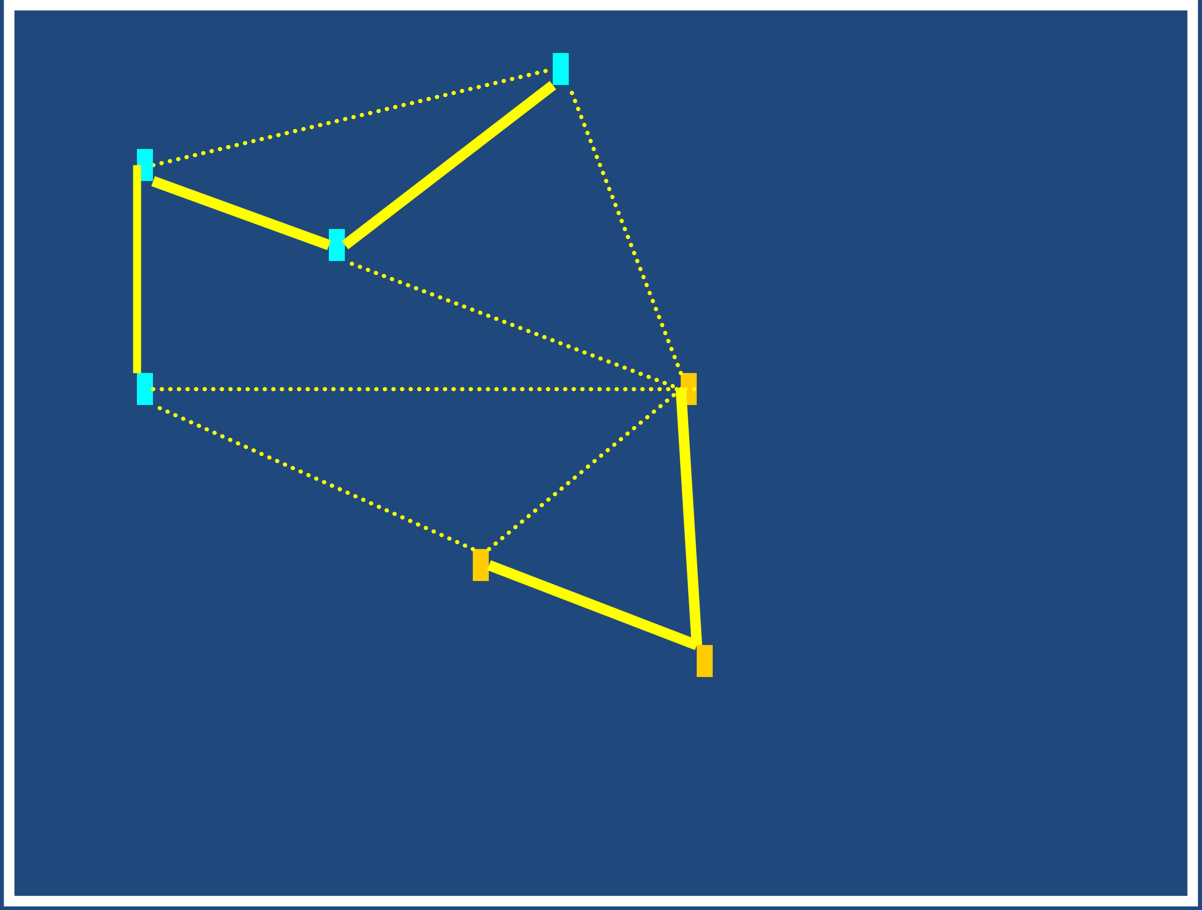
- Process the next shortest edge.
- The vertices connected are in different sets
- Add the edge to MST
- Merge the vertices

Kruskal's MST Algorithm – Example Step 5



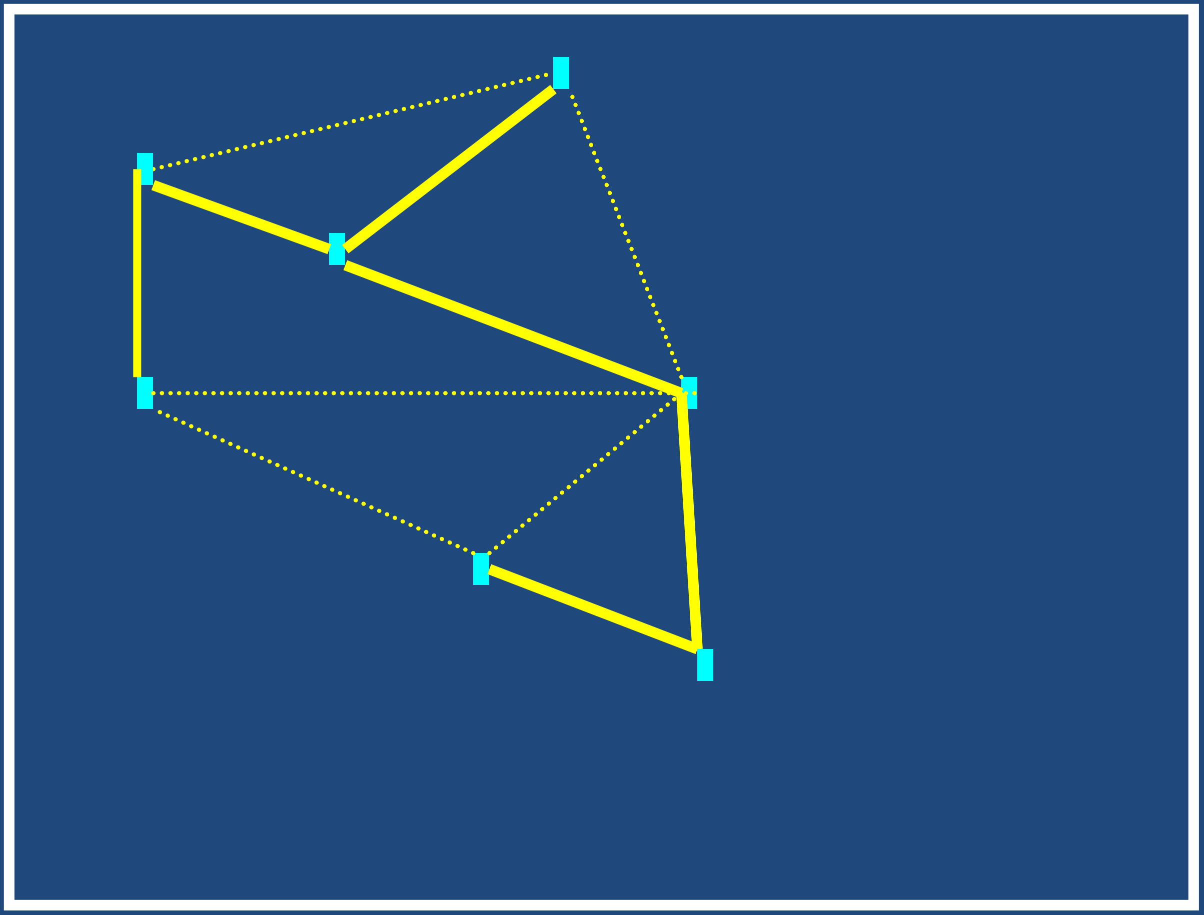
- Process the next shortest edge.
- The vertices connected are in the same set
- Skip the edge

Kruskal's MST Algorithm – Example Step 6



- Process the next shortest edge.
- The vertices connected are in different sets
- Add the edge to MST
- Merge the vertices

Kruskal's MST Algorithm – Example Step 7



- All vertices are connected
- MST edges are highlighted

Prim's MST Algorithm

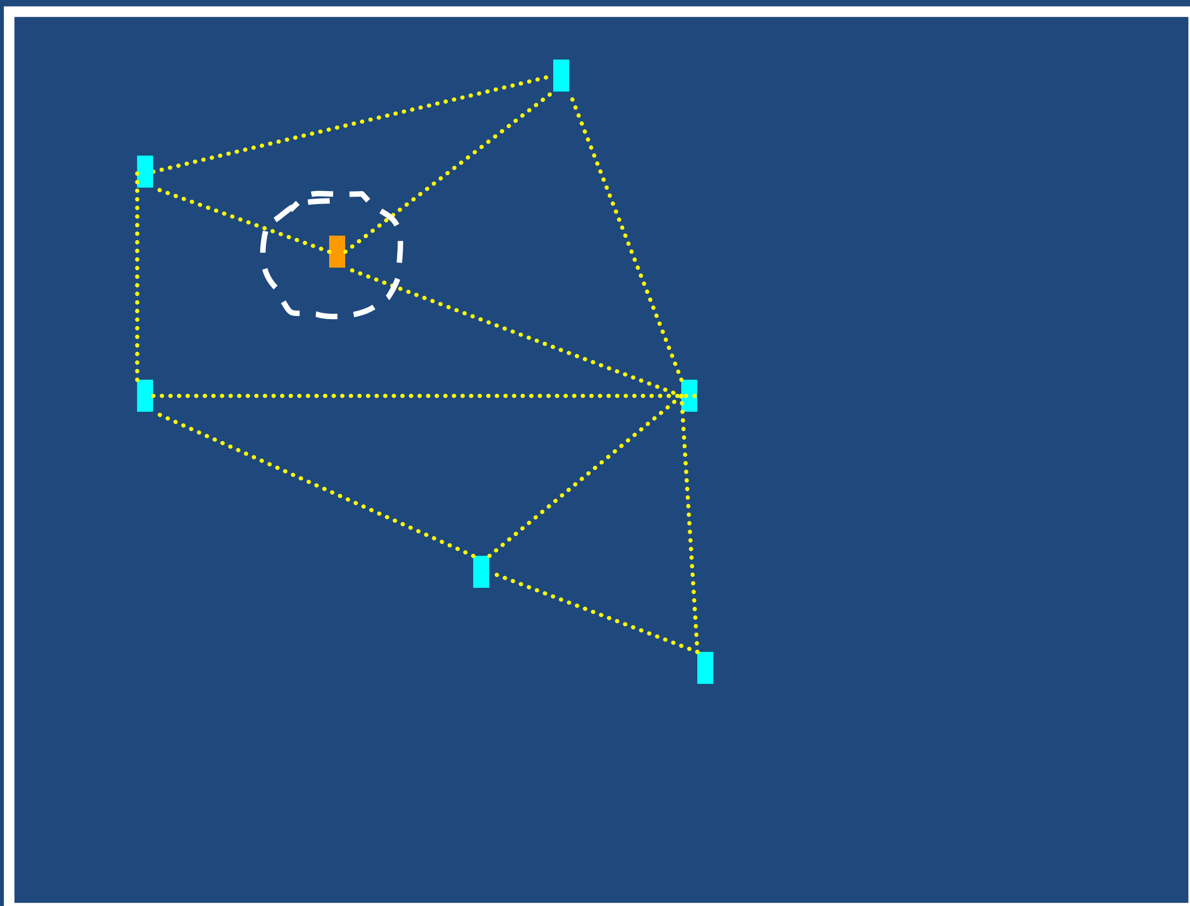
1. Initialize V_T and MST to be empty set

2. Pick a root vertex v in V (e.g. driver terminal)
3. Add v to V_T

4. While V_T is not equal to V
   1. Find edge e = (v1, v2) such that
      1. v1 is in V_T
      2. v2 is NOT in V_T
      3. weight of e is minimum
   2. Add e to MST
   3. Add v2 to V_T

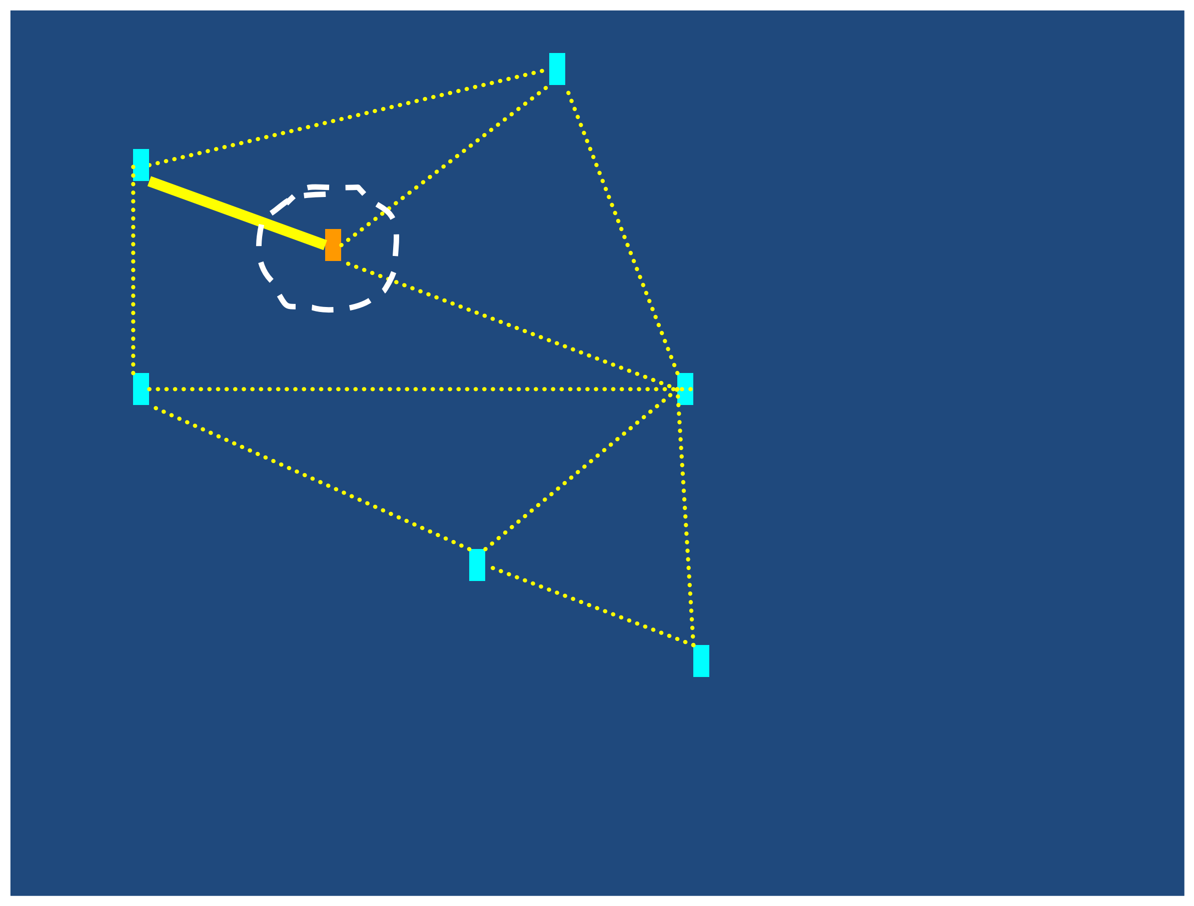
5. Return MST

Prim's MST Algorithm - Example



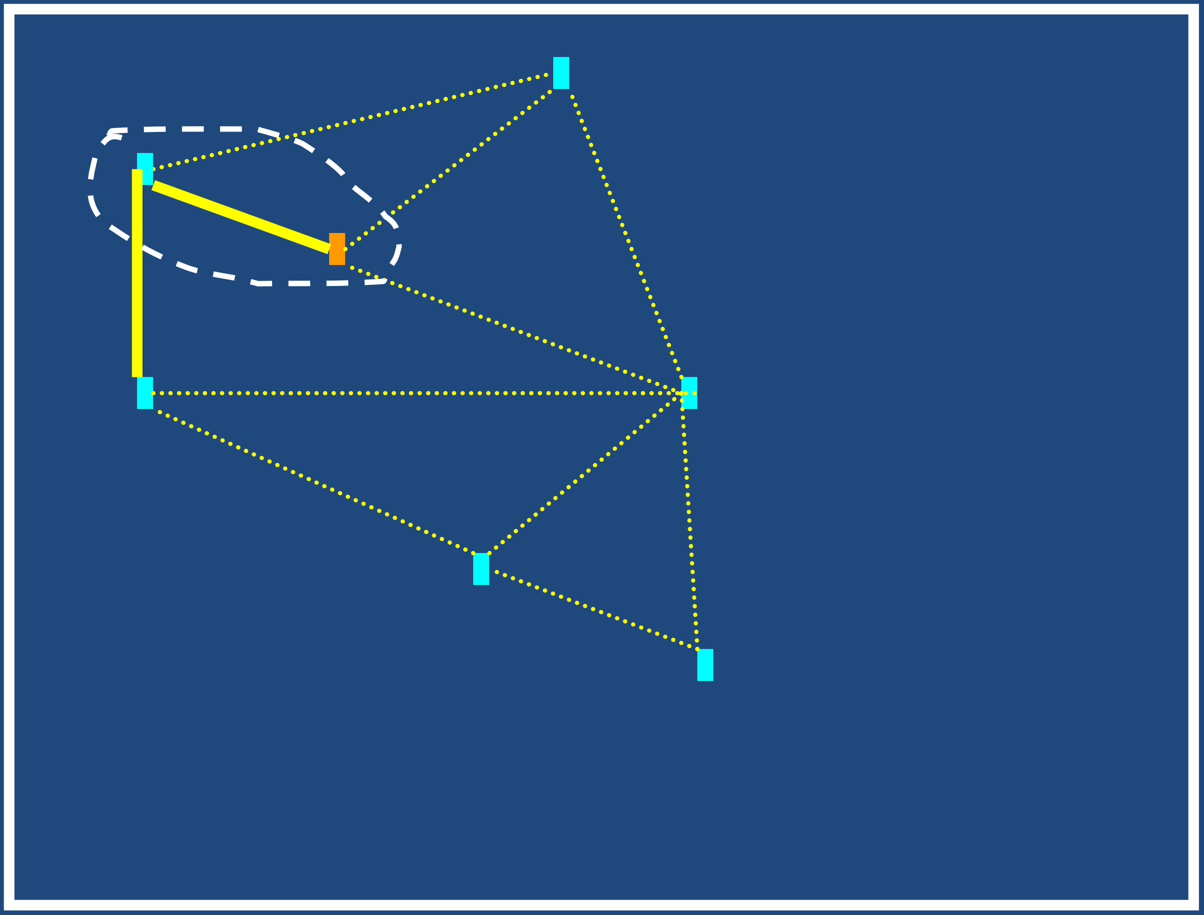
- Initially, V_T contains the root vertex (e.g. driver)

Prim's MST Algorithm - Example



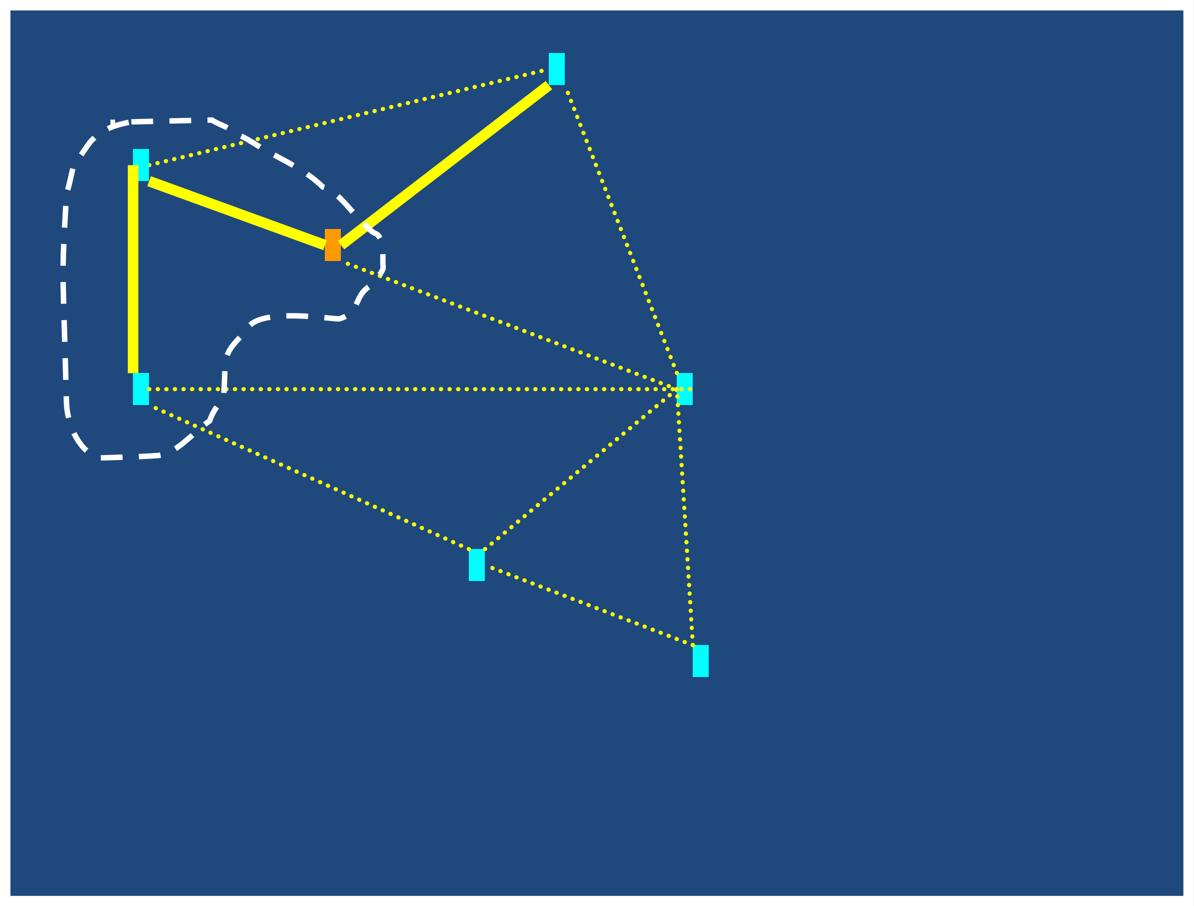
- Pick the shortest edge between V_T and $V - V_T$
- Add that edge to MST
- Expand V_T

Prim's MST Algorithm - Example



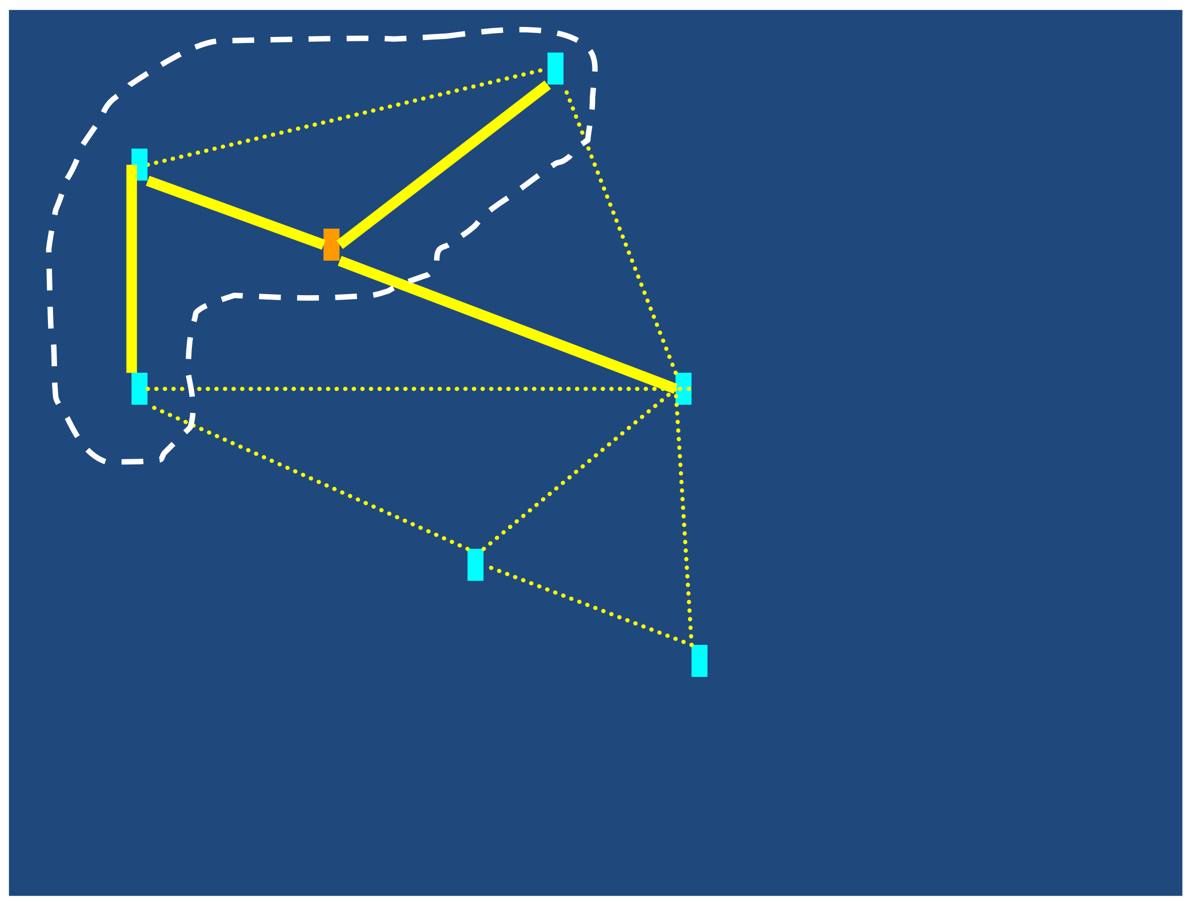
- Pick the shortest edge between V_T and $V - V_T$
- Add that edge to MST
- Expand V_T

Prim's MST Algorithm - Example



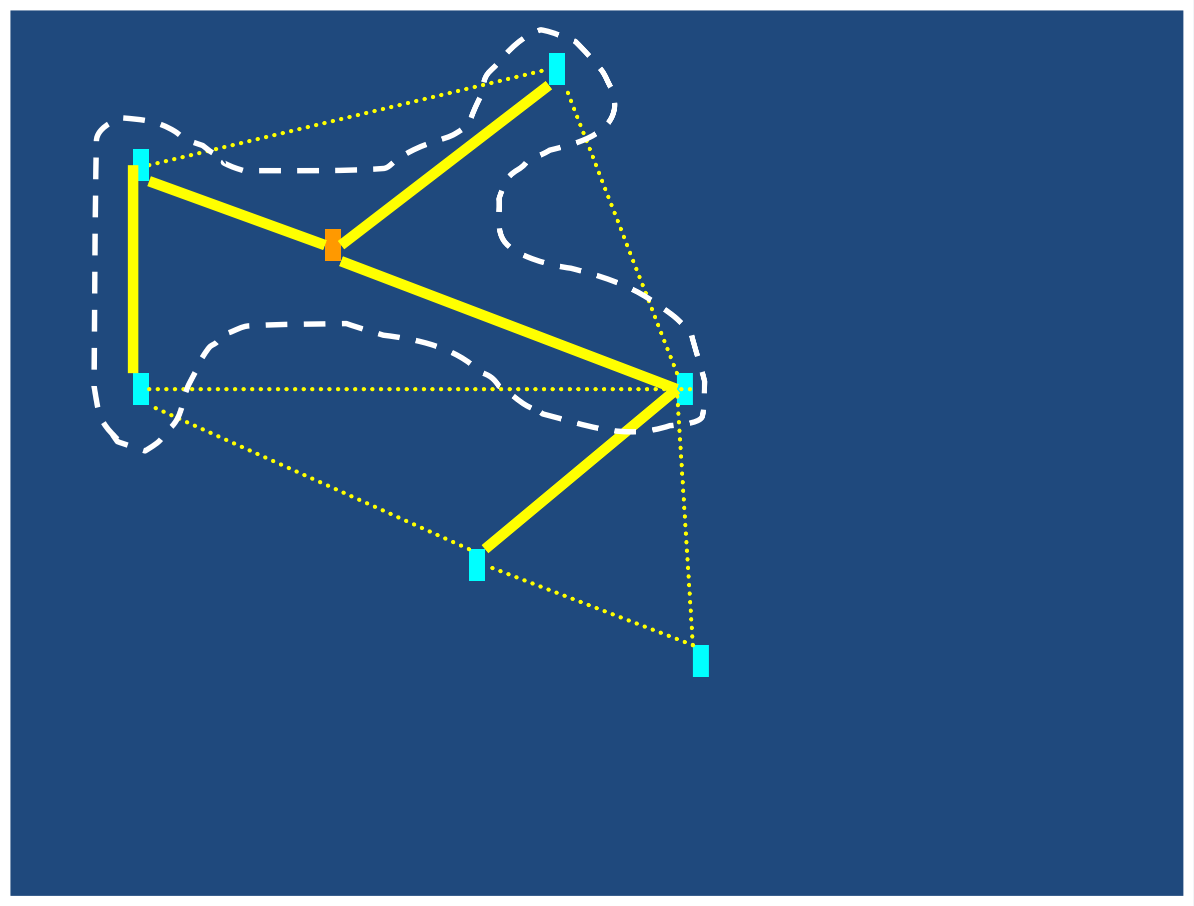
- Pick the shortest edge between V_T and $V - V_T$
- Add that edge to MST
- Expand V_T

Prim's MST Algorithm - Example



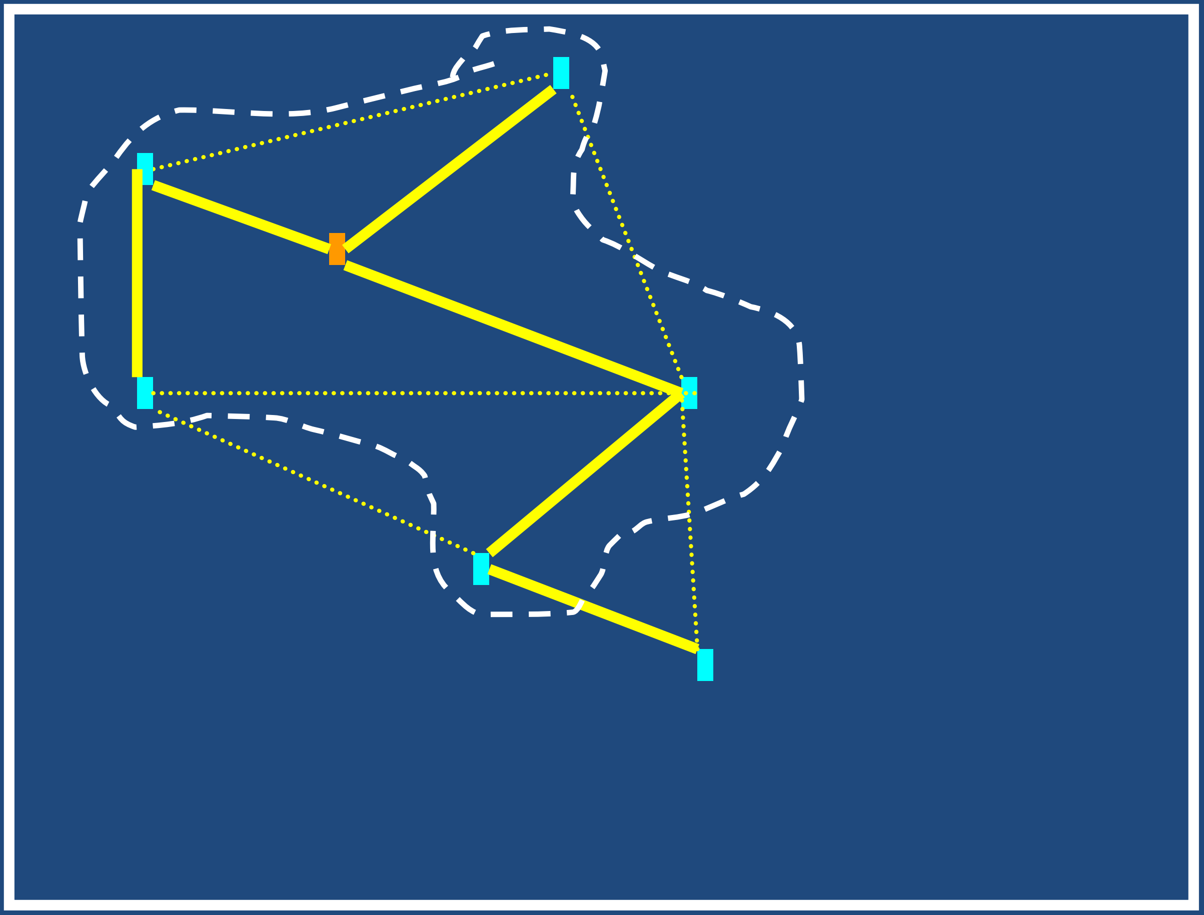
- Pick the shortest edge between V_T and $V - V_T$
- Add that edge to MST
- Expand V_T

Prim's MST Algorithm - Example



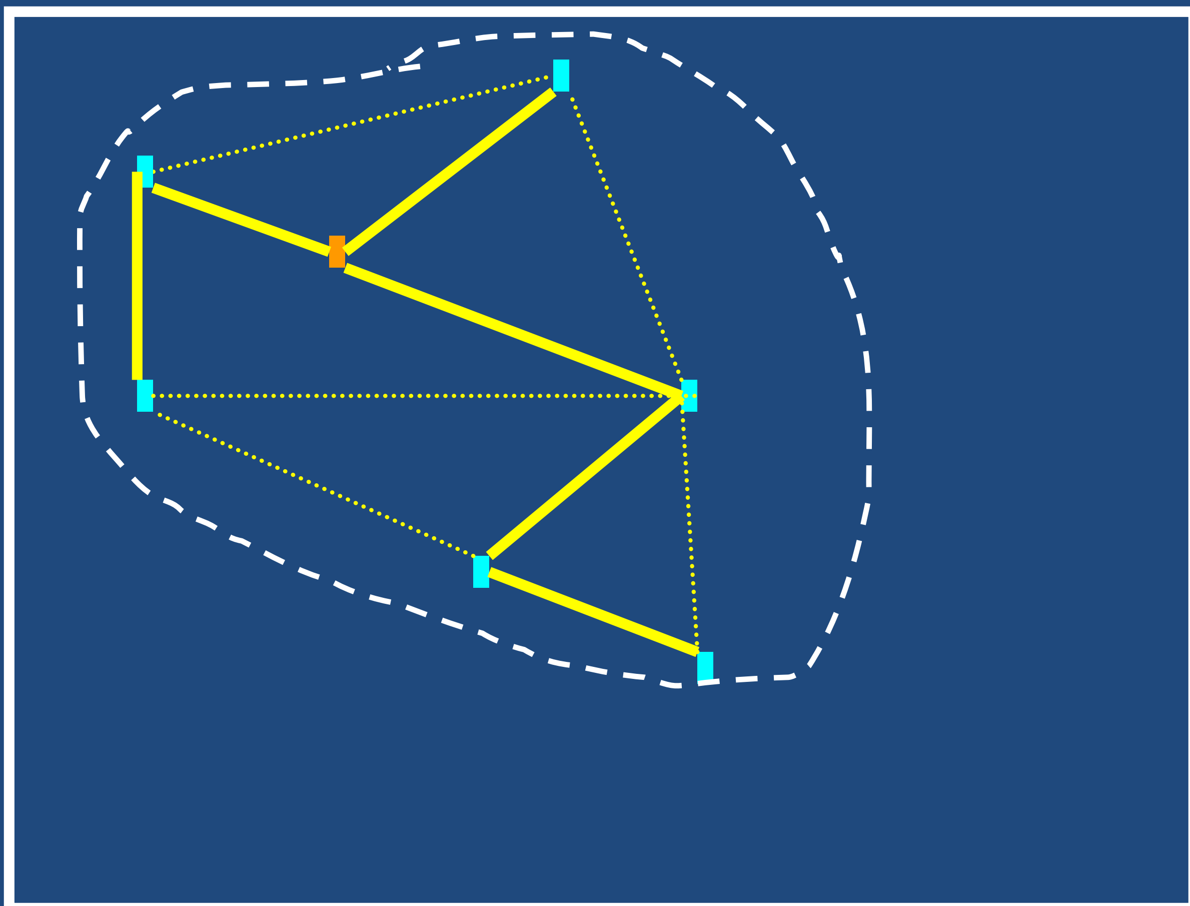
- Pick the shortest edge between V_T and $V - V_T$
- Add that edge to MST
- Expand V_T

Prim's MST Algorithm - Example



- Pick the shortest edge between V_T and $V - V_T$
- Add that edge to MST
- Expand V_T

Prim’s MST Algorithm - Example



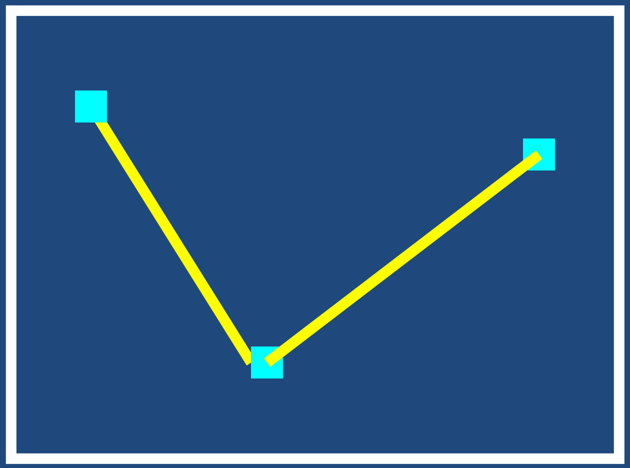
- All vertices are included in V_T
- MST edges are highlighted

MST – Summary

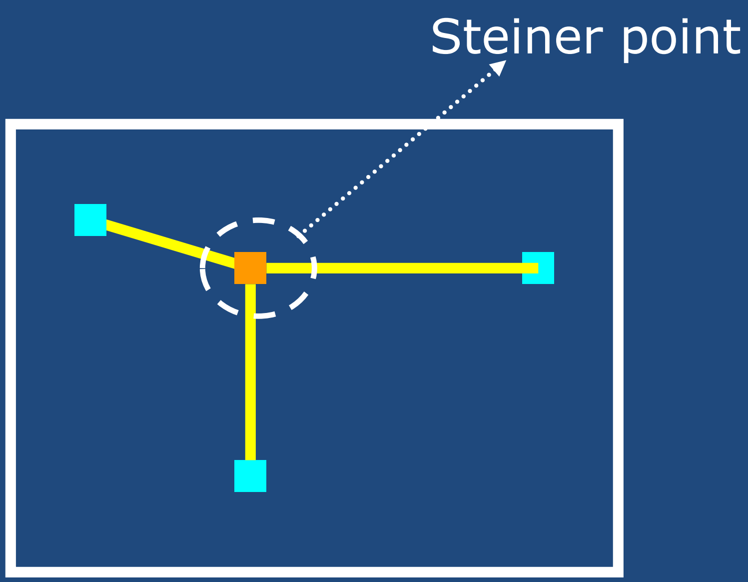
- Find the min-cost edge set that connects a given vertex set.
- Possible to solve it optimally in O(ElogE) time
  - Kruskal's algorithm
  - Prim's algorithm
- In general Prim's algorithm is better to control timing tradeoffs because we expand a wavefront from the driver.

Steiner Trees

- Similar to MSTs, but:
  - Extra intermediate vertices can be added to reduce wirelength.



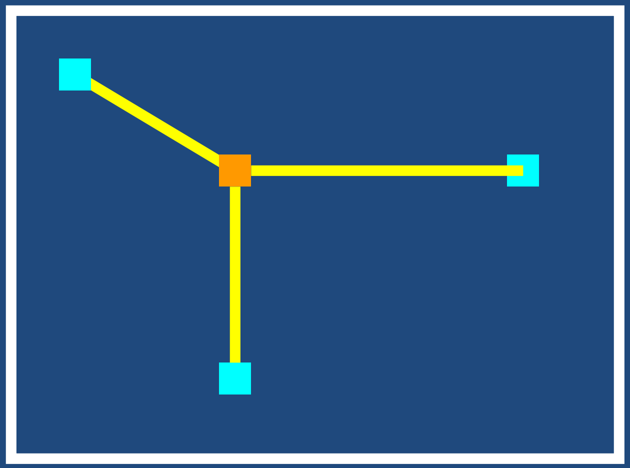
MST



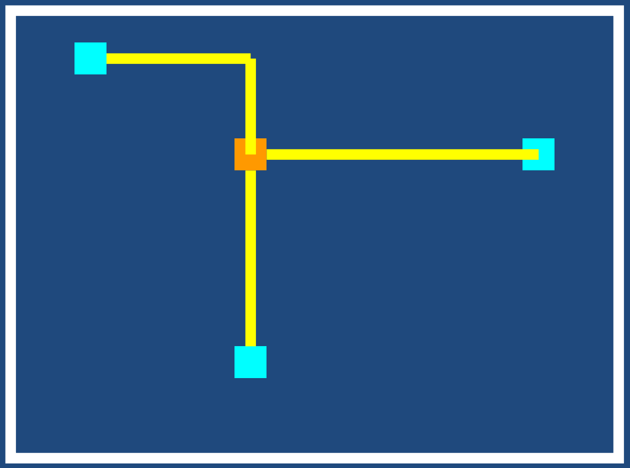
Steiner tree

Rectilinear Steiner Trees

- Steiner trees of which edges are all Manhattan
  - i.e. The routing of the slanted edges are all pre-determined



Steiner tree



Rectilinear Steiner tree

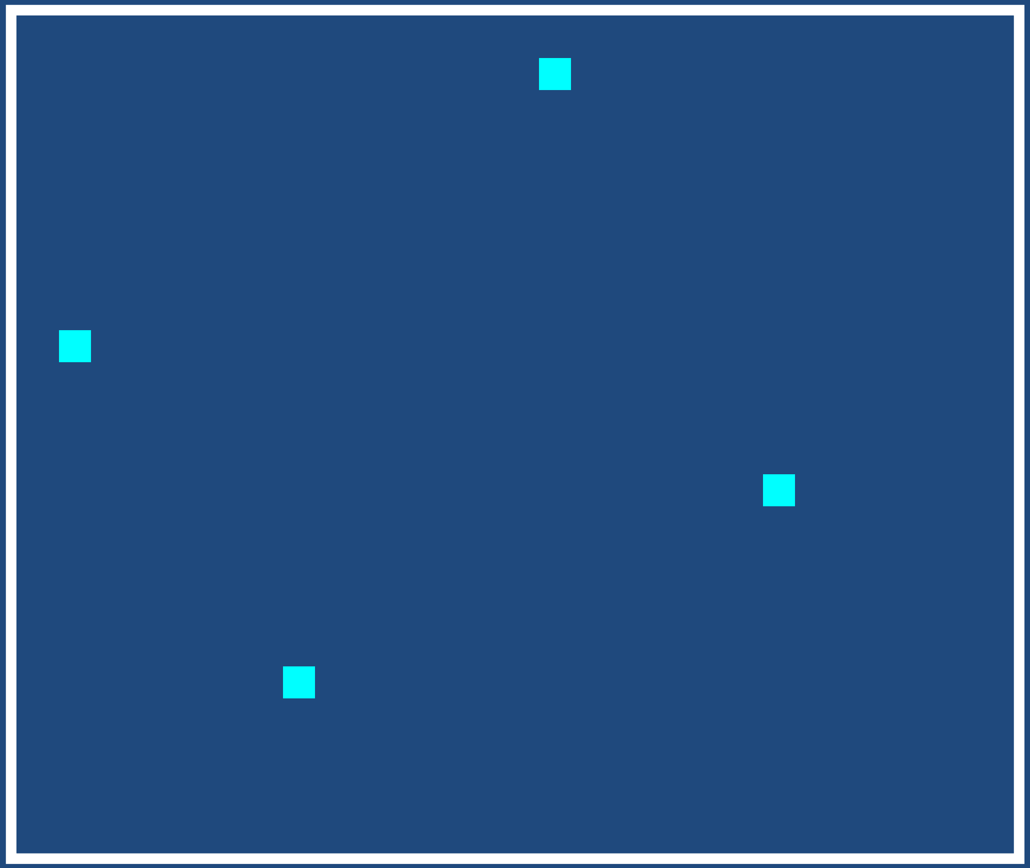
Steiner Tree Algorithms

- Steiner tree problem is NP-complete
  - Most likely there's no polynomial time optimal algorithm
  - Note: MST problem can be solved optimally in O(ElogE)

- Many Steiner tree heuristics
  - Iteratively add Steiner points to an MST
  - Route each edge of MST allowing Steiner points be created in the process.
  - Exponential time algorithms: Based on ILP, SAT, SMT solvers
  - A popular and practical algorithm: FLUTE

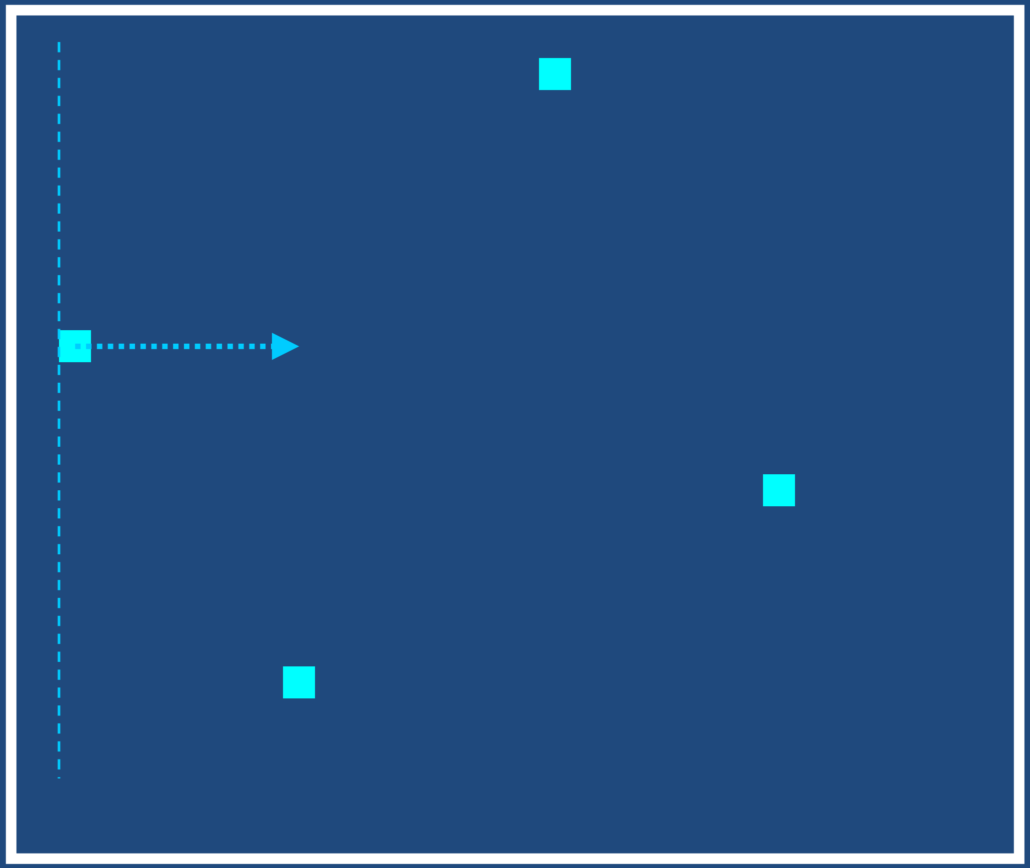
    C. Chu et al., "FLUTE: Fast Lookup Table Based Rectilinear Steiner Minimal Tree Algorithm for VLSI Design", IEEE Trans. On CAD, Jan 2008.

FLUTE for Steiner Tree Generation Example



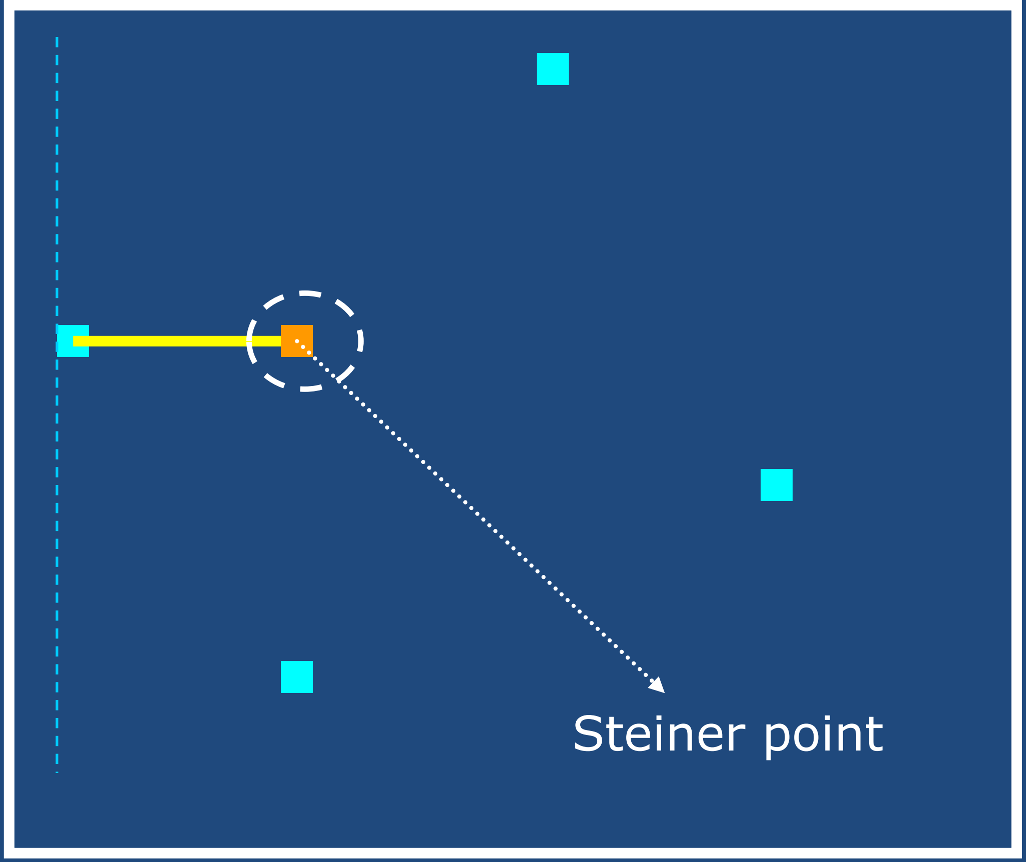
- “Press” terminals from each side

FLUTE for Steiner Tree Generation Example



- Press from left
- From the leftmost terminal to the second leftmost one.

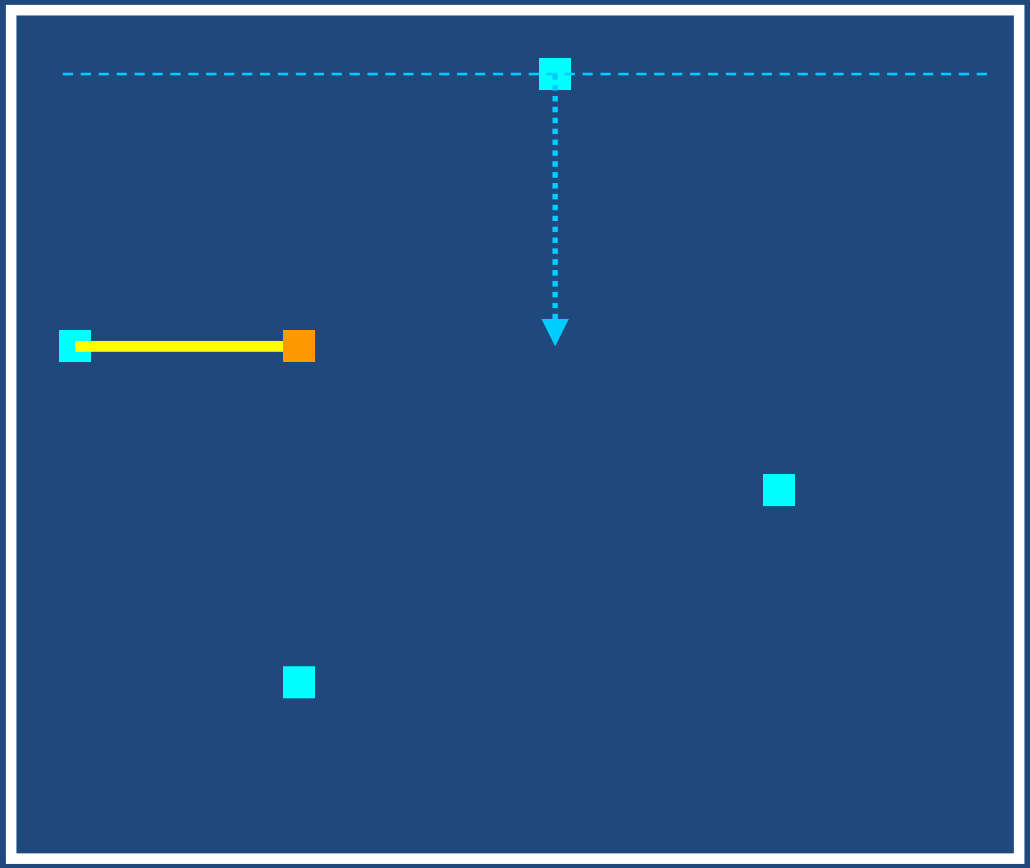
FLUTE for Steiner Tree Generation Example



- Create a Steiner edge corresponding to pressing edge
- Create a Steiner point at the new location

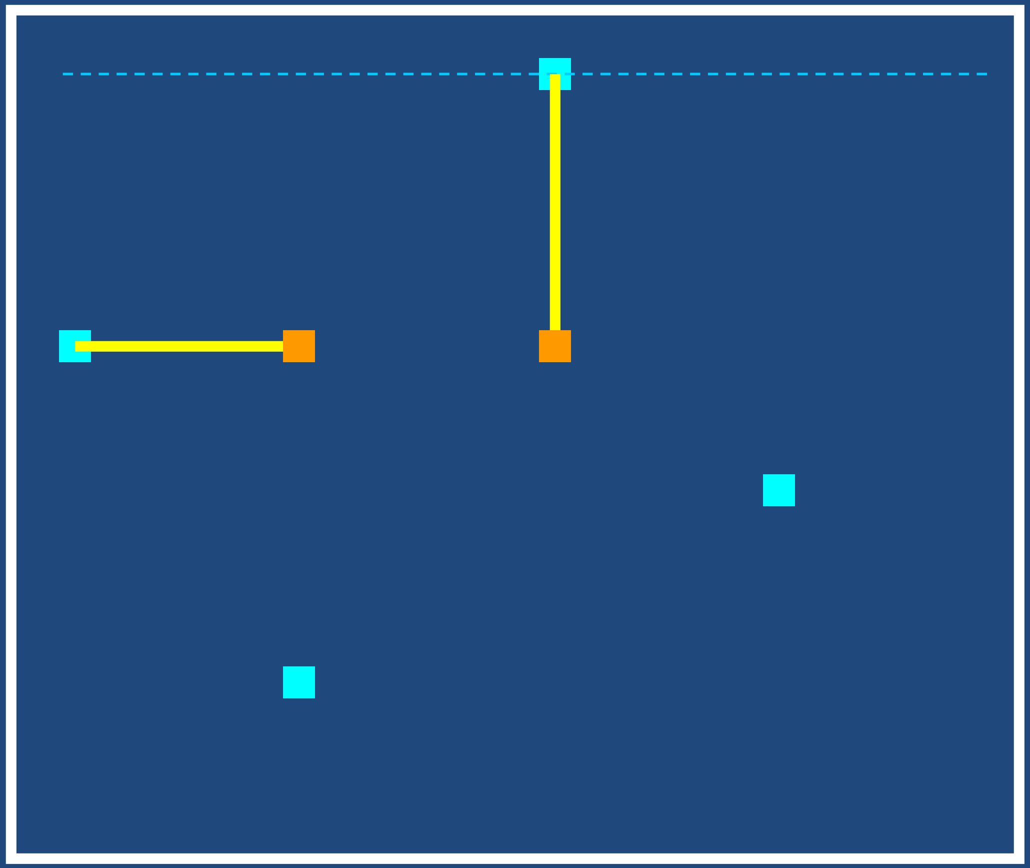
It is proven that pressing maintains optimality of Steiner tree.

FLUTE for Steiner Tree Generation Example



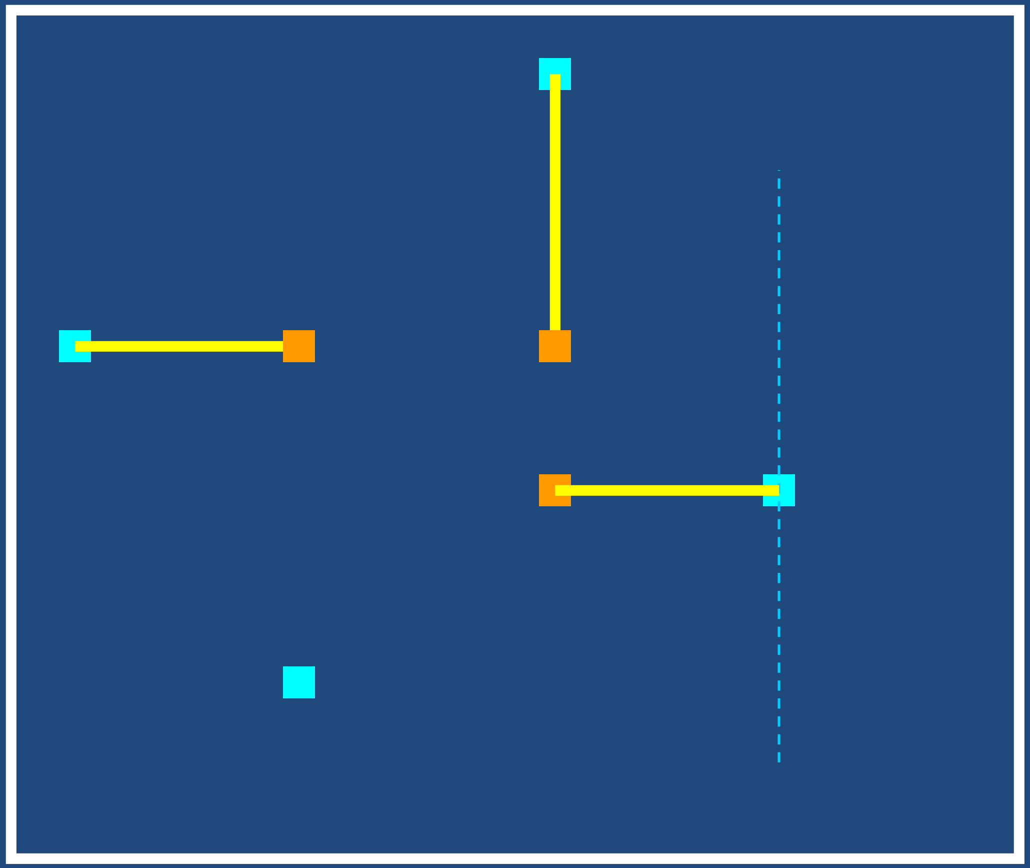
- Press from top

FLUTE for Steiner Tree Generation Example



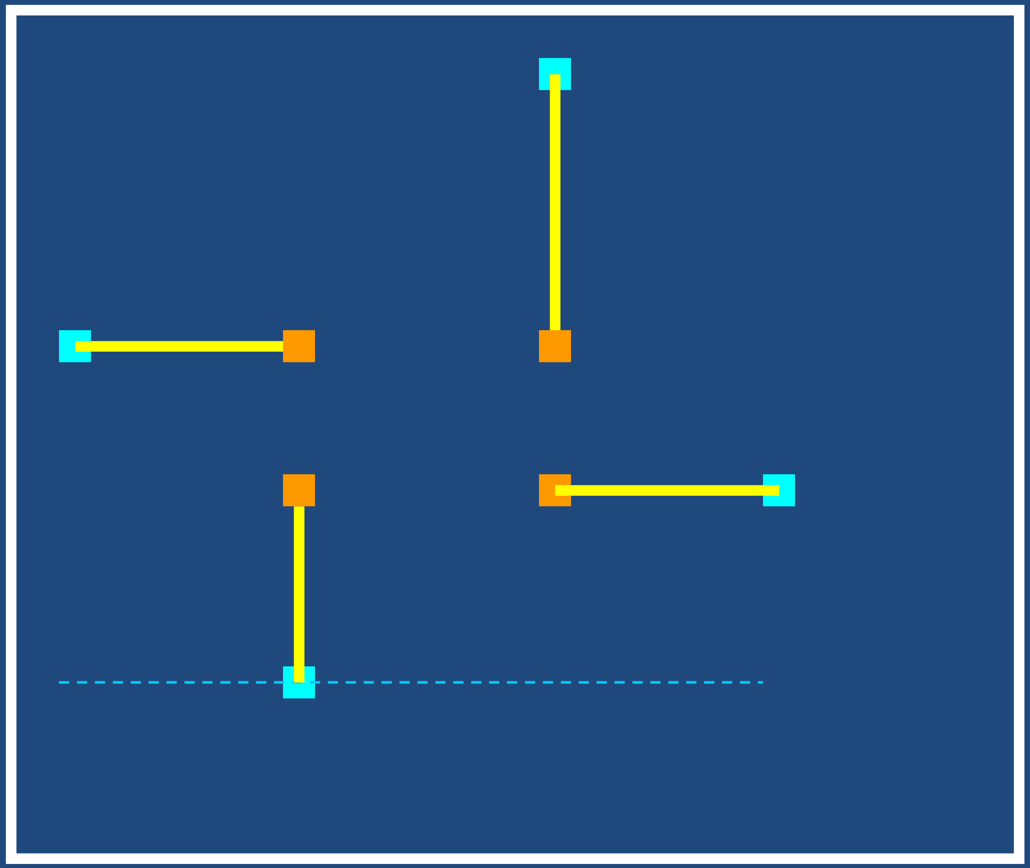
- Press from top

FLUTE for Steiner Tree Generation Example



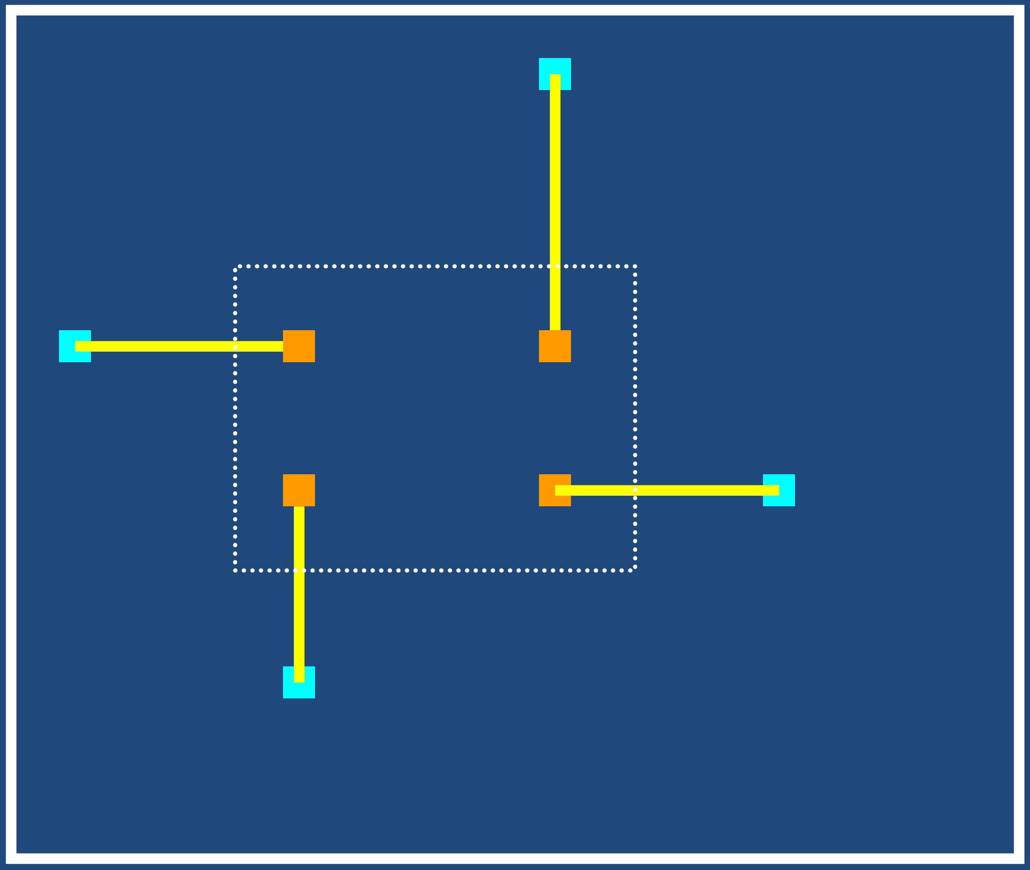
- Press from right

FLUTE for Steiner Tree Generation Example



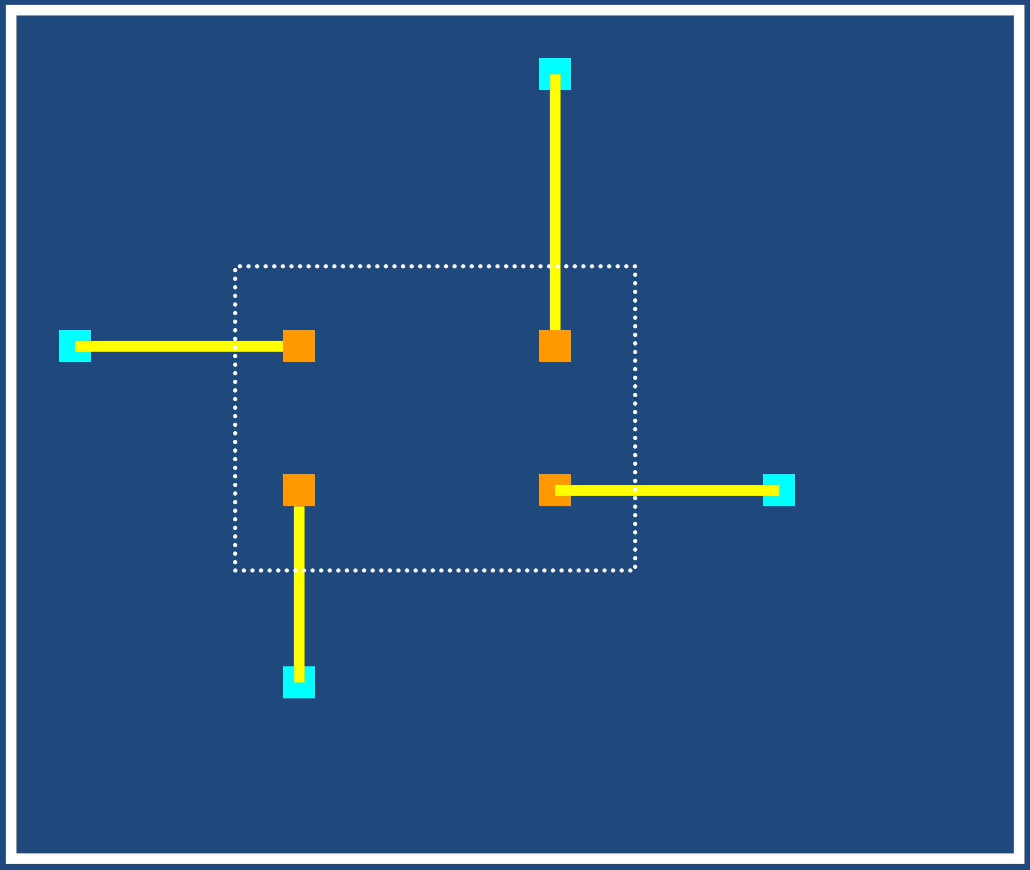
- Press from bottom

FLUTE for Steiner Tree Generation Example



- Problem is reduced to the rectangle at the center.
- How to connect these 4 Steiner (orange) points?

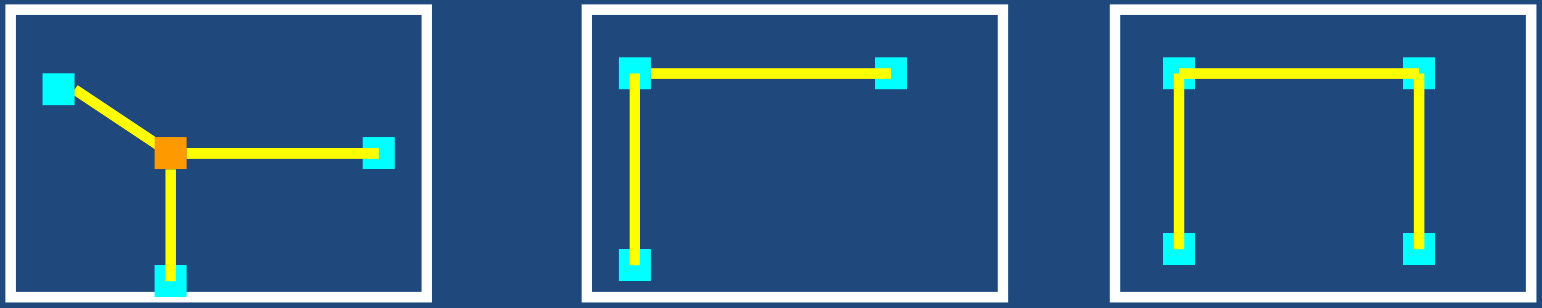
FLUTE for Steiner Tree Generation Example



- FLUTE pre-computes all Steiner tree solutions for such rectangles up to 10 terminals.
- After pressing, if there are less than 10 nodes, returns the solution from database.

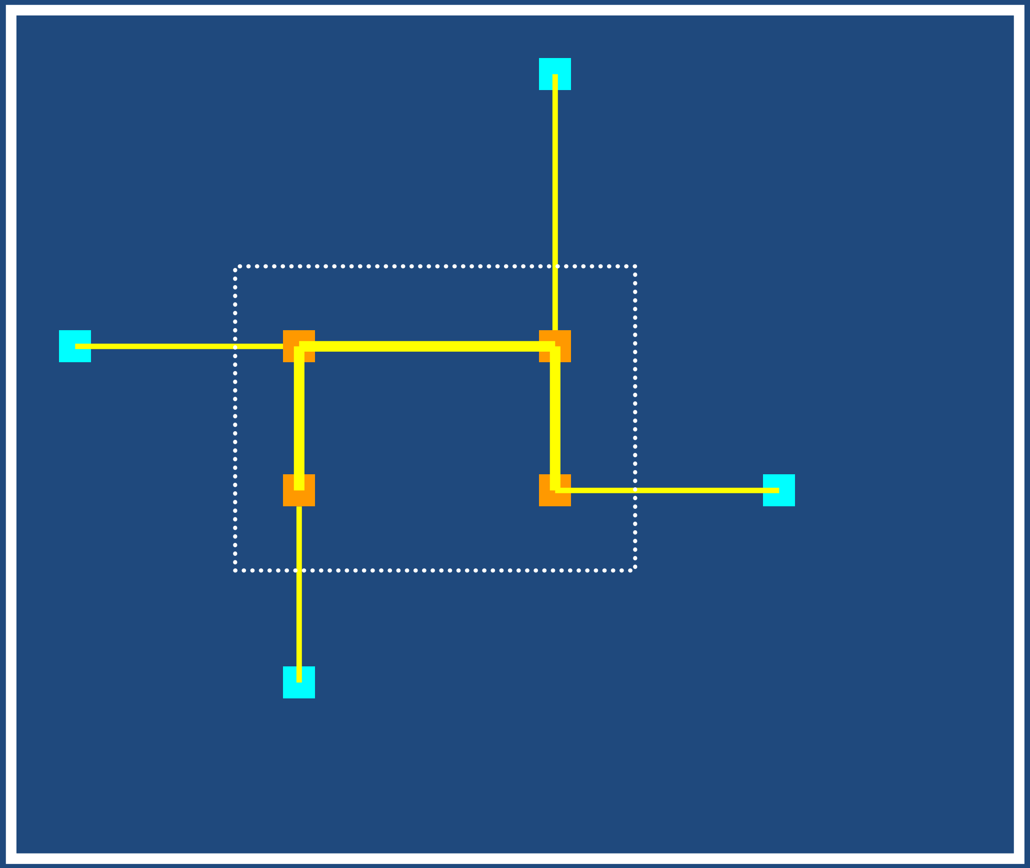
FLUTE for Steiner Tree Generation

- The solutions for simple problems are stored in a database.
- After pressing operations, if the problem is turned into one of those in the database, the best solution in the database is returned.
- If not in the database, use reduction heuristics.



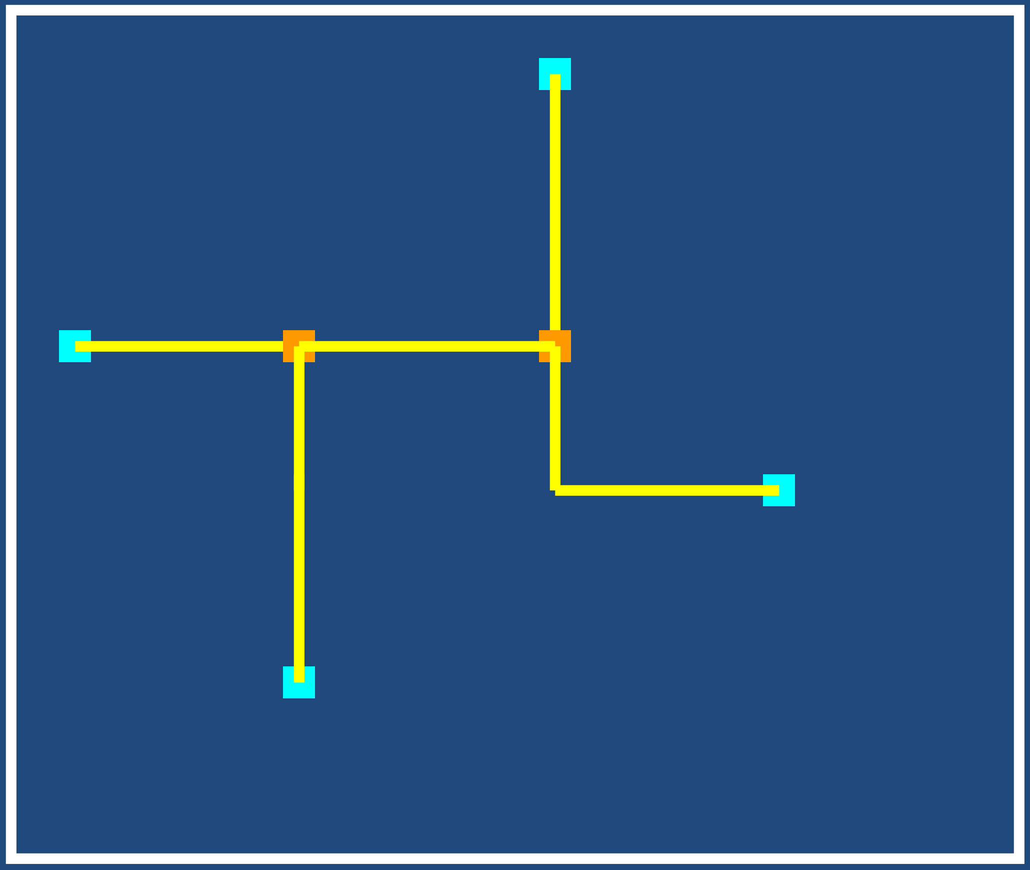
Canonical solutions stored in database

FLUTE for Steiner Tree Generation Example



- Solution inside the center rectangle is chosen from the database.

FLUTE for Steiner Tree Generation Example

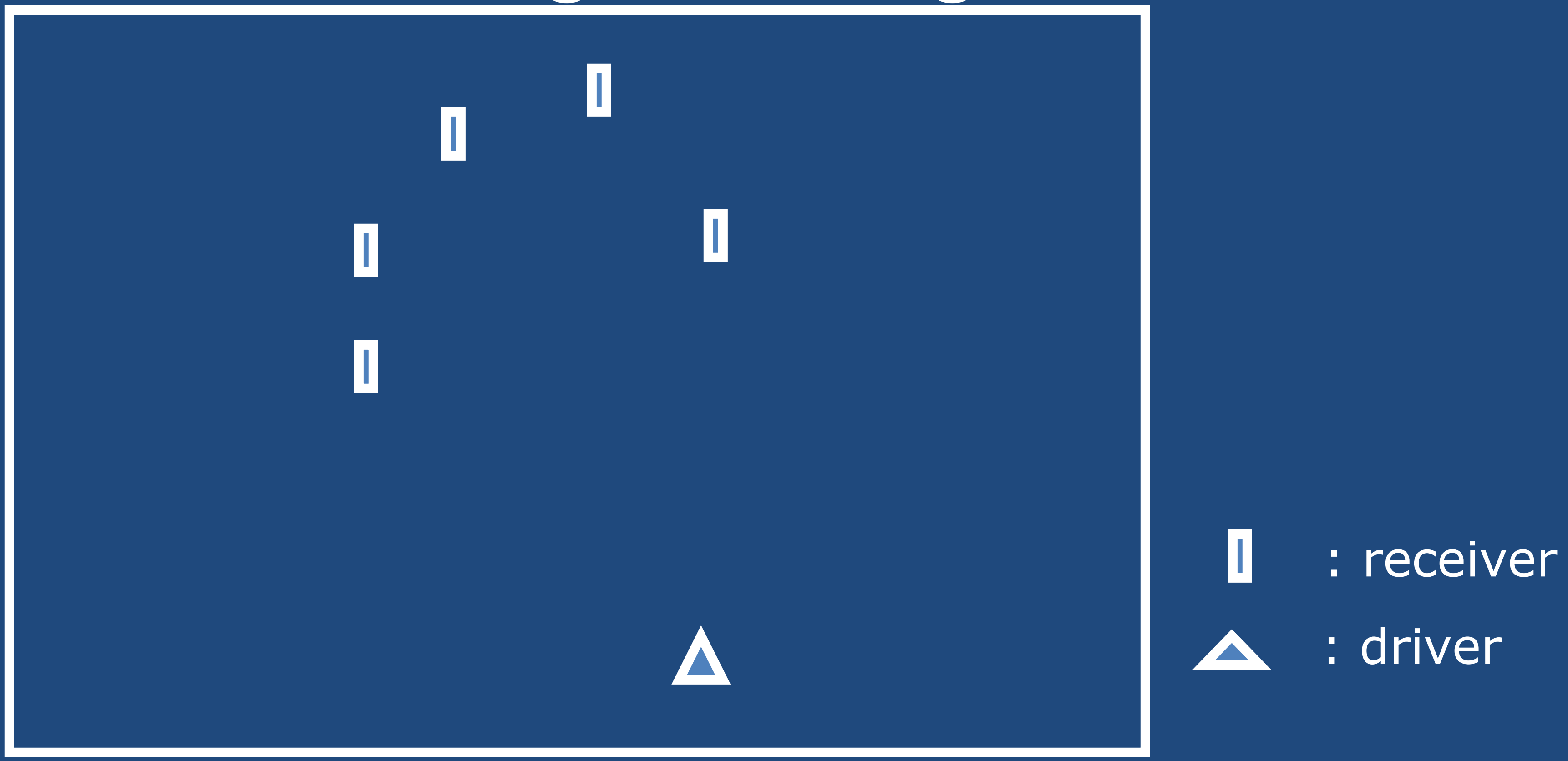


- Final rectilinear Steiner tree

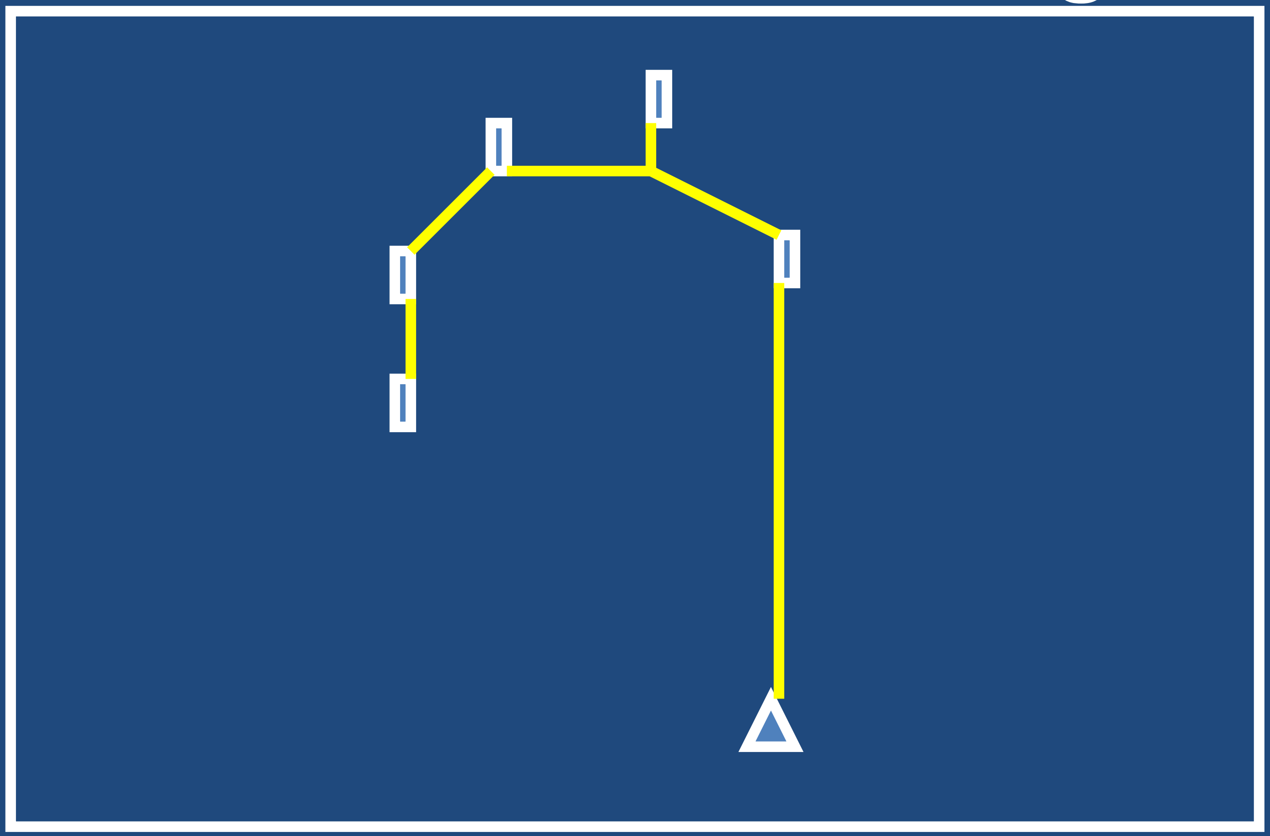
Cost Metrics

- Many tradeoffs to consider for routing topologies
- Topology with best wirelength can have poor timing
- Topology with best wirelength and timing may not be routable

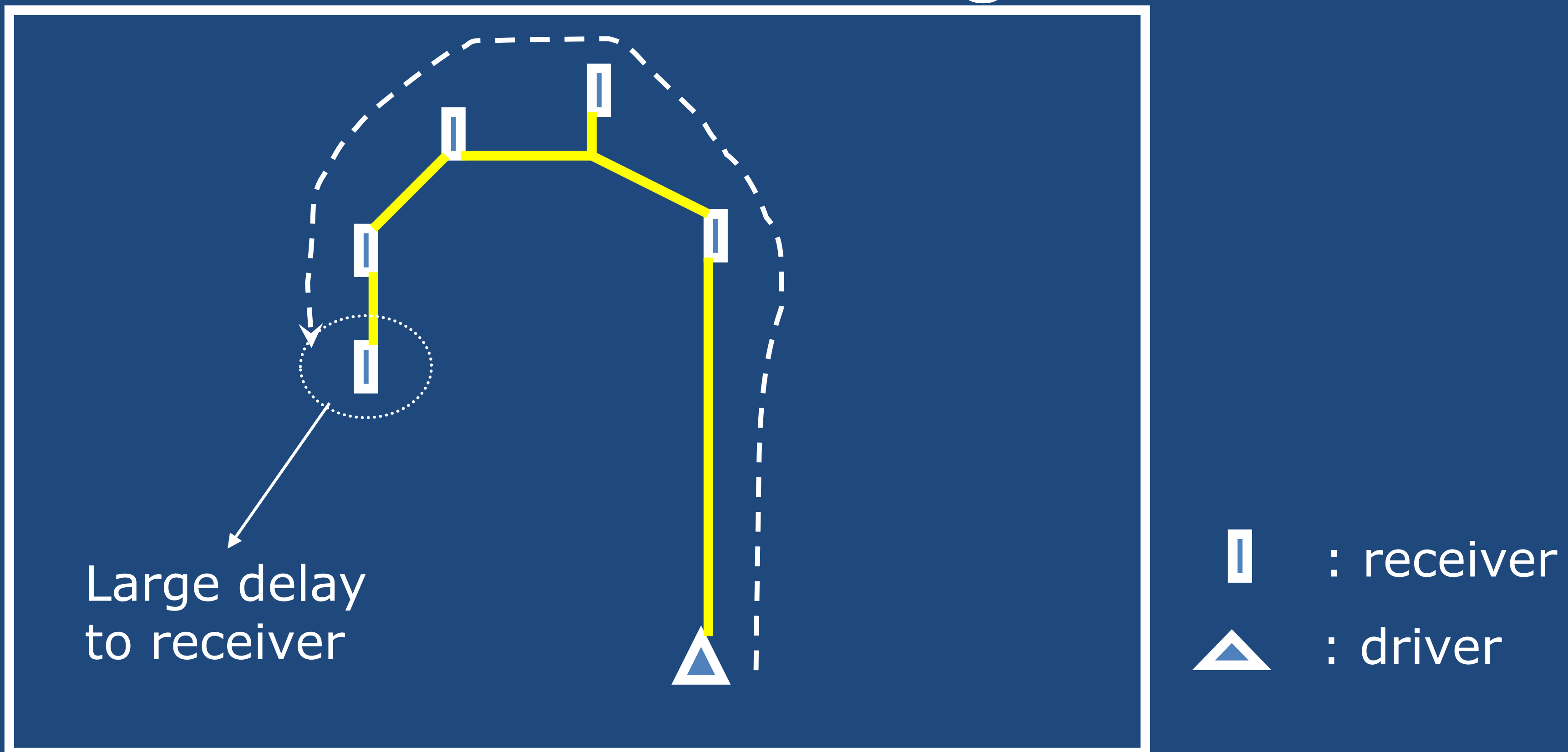
Example
Wirelength/Timing Tradeoff



Example
Min Wirelength



Example
Min Wirelength



Example
Better Timing – Worse Wirelength