CS425: Algorithms for Web Scale Data

Lecture 9: Recommender Systems: Latent Factor Models & Netflix Challenge

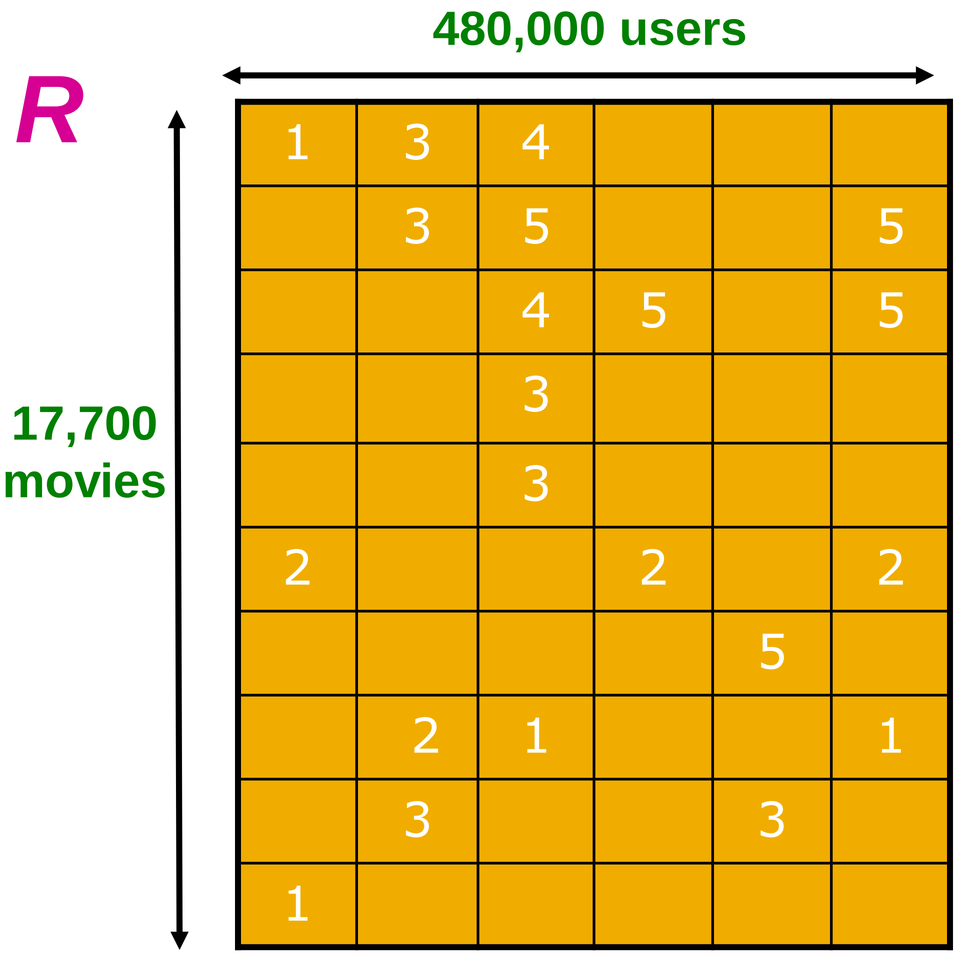
Most of the slides are from the Mining of Massive Datasets book.
These slides have been modified for CS425. The original slides can be accessed at: www.mmds.org

The Netflix Prize

- Training data
  - 100 million ratings, 480,000 users, 17,770 movies
  - 6 years of data: 2000-2005
- Test data
  - Last few ratings of each user (2.8 million)
  - **Evaluation criterion:** Root Mean Square Error (RMSE) $= \frac{1}{|R|}\sqrt{\sum_{(i,x)\in R}(\hat{r}_{xi} - r_{xi})^2}$
  - **Netflix's system RMSE: 0.9514**
- Competition
  - 2,700+ teams
  - **$1 million** prize for 10% improvement on Netflix

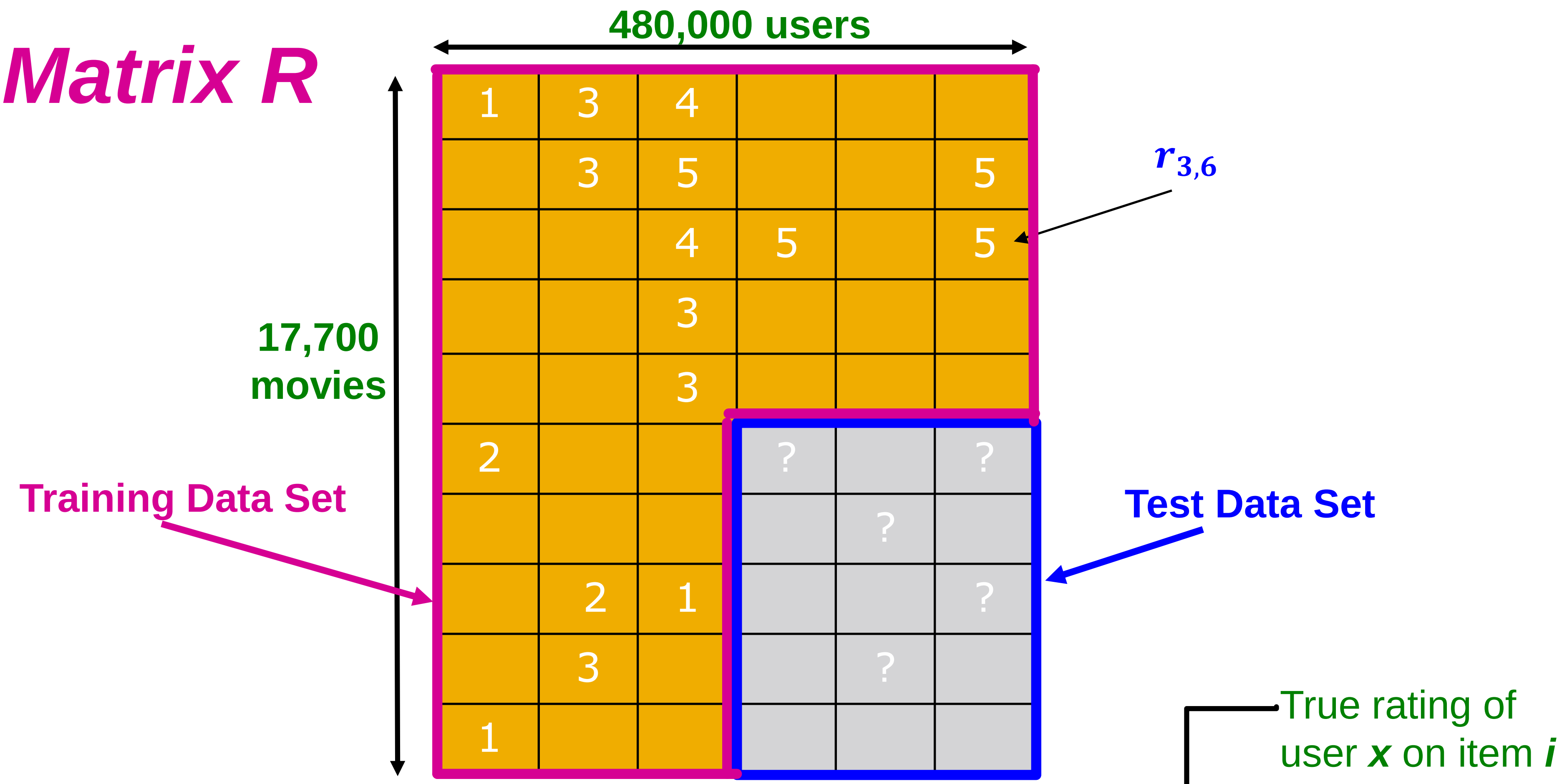
The Netflix Utility Matrix *R*

Matrix R



1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			2		2
				5	
	2	1			1
	3			3	
1					

Utility Matrix *R*: Evaluation



1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			?		?
				?	
	2	1			?
	3			?	
1					

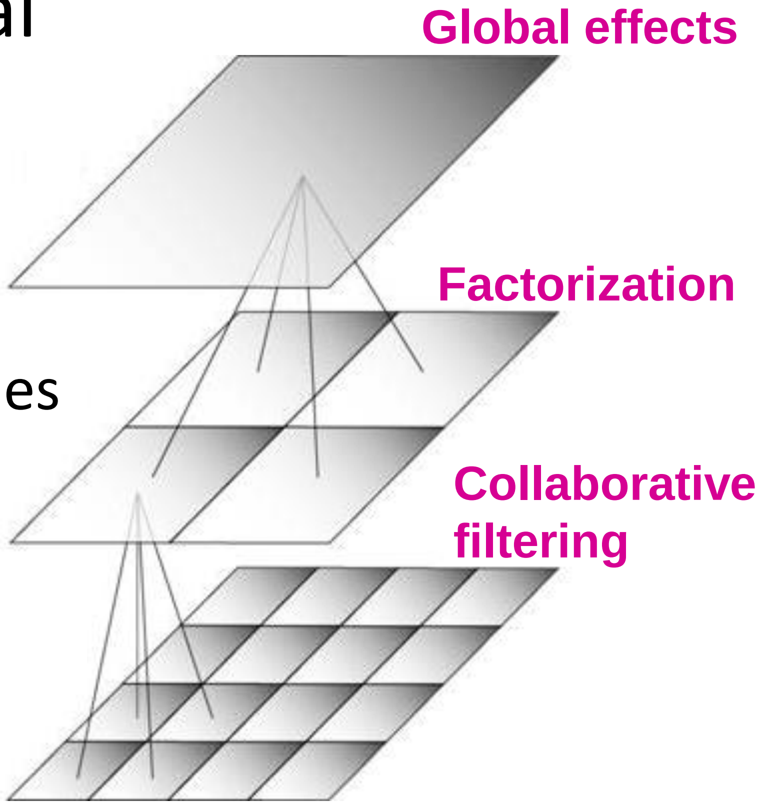
True rating of user ***x*** on item ***i***

$$\textbf{RMSE} = \frac{1}{|R|}\sqrt{\sum_{(i,x)\in R}(\hat{r}_{xi} - r_{xi})^2}$$

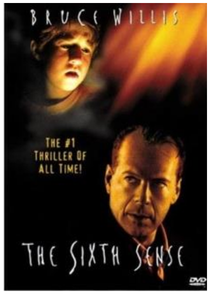
Predicted rating

BellKor Recommender System

- **The winner of the Netflix Challenge!**
- **Multi-scale modeling of the data:** Combine top level, "regional" modeling of the data, with a refined, local view:
  - **Global:**
    - Overall deviations of users/movies
  - **Factorization:**
    - Addressing "regional" effects
  - **Collaborative filtering:**
    - Extract local patterns



Modeling Local & Global Effects

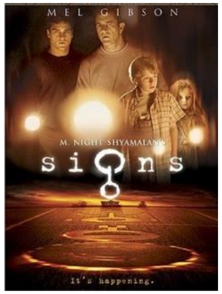


- **Global:**
  - Mean movie rating: **3.7 stars**
  - *The Sixth Sense* is **0.5** stars above avg.
  - Joe rates **0.2** stars below avg.

    ⇒ **Baseline estimation:**

    ***Joe* will rate *The Sixth Sense* 4 stars**
- **Local neighborhood (CF/NN):**
  - *Joe* didn't like related movie *Signs*
  - ⇒ **Final estimate:**

    ***Joe* will rate *The Sixth Sense* 3.8 stars**



Recap: Collaborative Filtering (CF)

- **Earliest and most popular collaborative filtering method**
- Derive unknown ratings from those of "**similar**" movies (item-item variant)
- Define **similarity measure s_{ij}** of items ***i*** and ***j***
- Select ***k***-nearest neighbors, compute the rating
    - ***N(i; x):*** items most similar to ***i*** that were rated by ***x***

$$\hat{r}_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

s_{ij}… similarity of items ***i*** and ***j***
r_{xj}…rating of user ***x*** on item ***j***
N(i;x)… set of items similar to item ***i*** that were rated by ***x***

Modeling Local & Global Effects

- **In practice we get better estimates if we model deviations:**

$$\hat{r}_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} s_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} s_{ij}}$$

baseline estimate for r_{xi}

$$b_{xi} = \mu + b_x + b_i$$

μ = overall mean rating

b_x = rating deviation of user x
= (*avg. rating of* user x) – μ

b_i = (*avg. rating of* movie i) – μ

Problems/Issues:

1) Similarity measures are "arbitrary"

2) Pairwise similarities neglect interdependencies among users

3) Taking a weighted average can be restricting

Solution: Instead of s_{ij} use w_{ij} that we estimate directly from data

Idea: Interpolation Weights w_{ij}

- Use a **weighted sum** rather than **weighted avg.**:

$$\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij}\left(r_{xj} - b_{xj}\right)$$

- **A few notes:**
  - $\boldsymbol{N(i;x)}$ … set of movies rated by user ***x*** that are similar to movie ***i***
  - $\boldsymbol{w_{ij}}$ is the interpolation weight (some real number)
    - We allow: $\boldsymbol{\sum_{j \in N(i,x)} w_{ij} \neq 1}$
  - $\boldsymbol{w_{ij}}$ models interaction between pairs of movies (it does not depend on user ***x***)

Idea: Interpolation Weights w_{ij}

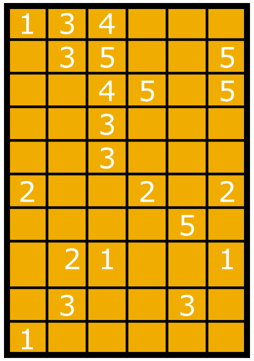
- $\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i,x)} w_{ij}(r_{xj} - b_{xj})$
- **How to set w_{ij}?**
  - Remember, error metric is: $\frac{1}{|R|}\sqrt{\sum_{(i,x)\in R}(\hat{r}_{xi} - r_{xi})^2}$

    or equivalently **SSE:** $\boldsymbol{\sum_{(i,x)\in R}(\hat{r}_{xi} - r_{xi})^2}$
  - Find w_{ij} that minimize **SSE** on **training data!**
    - Models relationships between item ***i*** and its neighbors ***j***
  - w_{ij} can be **learned/estimated** based on ***x*** and all other users that rated ***i***

Why is this a good idea?

Recommendations via Optimization

- **Goal:** Make good recommendations
  - Quantify goodness using **RMSE:** **Lower RMSE ⇒ better recommendations**
  - Want to make good recommendations on items that user has not yet seen. Can't really do this!
  - **Let's set build a system such that it works well on known (user, item) ratings** And **hope** the system will also predict well the **unknown ratings**



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		4	5		5
		3			
		3			
2			2		2
				5	
	2	1			1
	3			3	
1					

Recommendations via Optimization

- **Idea: Let's set values *w* such that they work well on known (user, item) ratings**
- **How to find such values *w*?**
- **Idea:** Define an objective function and solve the optimization problem

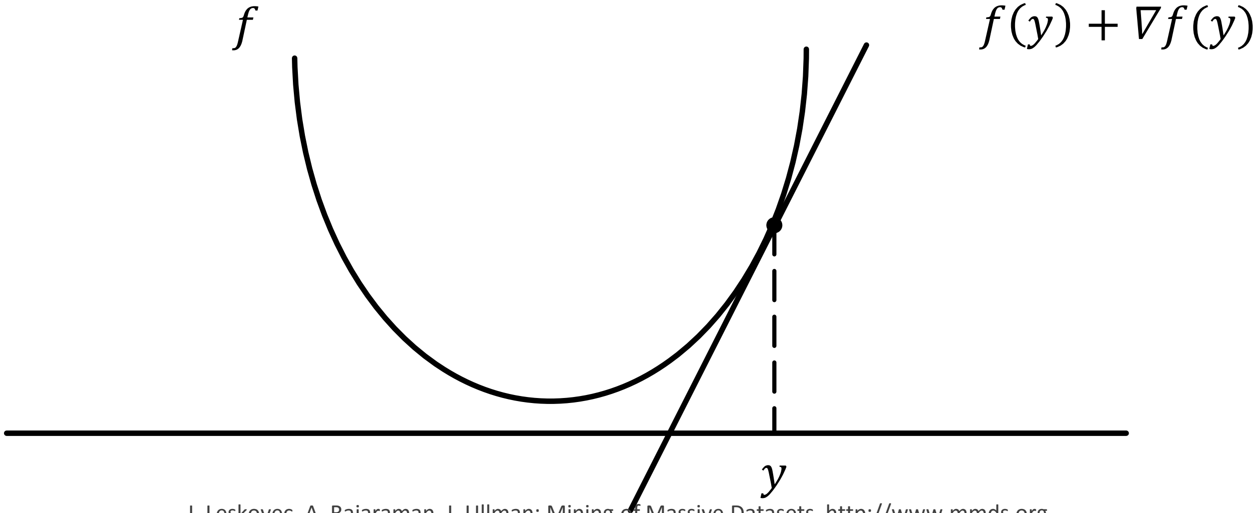
- Find $\boldsymbol{w_{ij}}$ that minimize **SSE** on **training data**!

$$J(w) = \sum_{x,i} \Bigg(\underbrace{\Bigg[b_{xi} + \sum_{j \in N(i;x)} w_{ij} \left(r_{xj} - b_{xj} \right) \Bigg]}_{\text{Predicted rating}} - \underset{\text{True rating}}{r_{xi}} \Bigg)^2$$

- Think of ***w*** as a vector of numbers

Detour: Minimizing a function

- **A simple way to minimize a function $f(x)$:**
  - Compute the derivative ∇f
  - **Start at some point y and evaluate $\nabla f(y)$**
  - **Make a step in the reverse direction of the gradient: $y = y - \nabla f(y)$**
  - **Repeat until converged**



Example: Formulation

- Assume we have a dataset with a single user x and items 0, 1, and 2. We are given all ratings, and we want to compute the weights w_{01}, w_{02}, and w_{03}.
- Rating estimate: $\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i,x)} w_{ij} \left(r_{xj} - b_{xj} \right)$

  Training dataset already has the correct r_{xi} values. We will use the estimation formula to compute the unknown weights w_{01}, w_{02}, and w_{03}.
- Optimization problem: Compute w_{ij} values to minimize:

$$\sum_{(\boldsymbol{i},\boldsymbol{x}) \in \boldsymbol{R}} (\hat{\boldsymbol{r}}_{\boldsymbol{xi}} - \boldsymbol{r}_{\boldsymbol{xi}})^{\mathbf{2}}$$

- Plug in the formulas:

$$\begin{aligned} \text{minimize} \quad \mathrm{J(w)} = \; & [b_{x0} + w_{01}(r_{x1} - b_{x1}) + w_{02}(r_{x2} - b_{x2}) \; - r_{x0}]^2 \\ & + [b_{x1} + w_{01}(r_{x0} - b_{x0}) + w_{12}(r_{x2} - b_{x2}) \; - r_{x1}]^2 \\ & + [b_{x2} + w_{02}(r_{x0} - b_{x0}) + w_{12}(r_{x1} - b_{x1}) \; - r_{x2}]^2 \end{aligned}$$

Example: Algorithm

Initialize unknown variables:

$$\mathbf{w}^{\mathbf{new}} = \begin{bmatrix} w_{01}^{new} \\ w_{02}^{new} \\ w_{12}^{new} \end{bmatrix} = \begin{bmatrix} w_{01}^{0} \\ w_{02}^{0} \\ w_{12}^{0} \end{bmatrix}$$

Iterate:

while $|\boldsymbol{w}^{new} - \boldsymbol{w}^{old}| > \varepsilon$

$$\boldsymbol{w}^{old} = \boldsymbol{w}^{new}$$

$$\boldsymbol{w}^{new} = \boldsymbol{w}^{old} - \eta \cdot \nabla J(\boldsymbol{w}^{old})$$

η is the learning rate (a parameter)

How to compute $\nabla J(\boldsymbol{w}^{old})$?

Example: Gradient-Based Update

$$\mathrm{J(w)} = [b_{x0} + w_{01}(r_{x1} - b_{x1}) + w_{02}(r_{x2} - b_{x2}) - r_{x0}]^2 + [b_{x1} + w_{01}(r_{x0} - b_{x0}) + w_{12}(r_{x2} - b_{x2}) - r_{x1}]^2 + [b_{x2} + w_{02}(r_{x0} - b_{x0}) + w_{12}(r_{x1} - b_{x1}) - r_{x2}]^2$$

$$\boldsymbol{\nabla J(w)} = \begin{bmatrix} \frac{\boldsymbol{\partial J(w)}}{\boldsymbol{\partial w_{01}}} \\ \frac{\boldsymbol{\partial J(w)}}{\boldsymbol{\partial w_{02}}} \\ \frac{\boldsymbol{\partial J(w)}}{\boldsymbol{\partial w_{12}}} \end{bmatrix}$$

$$\begin{bmatrix} w_{01}^{new} \\ w_{02}^{new} \\ w_{12}^{new} \end{bmatrix} = \begin{bmatrix} w_{01}^{old} \\ w_{02}^{old} \\ w_{12}^{old} \end{bmatrix} - \eta \begin{bmatrix} \frac{\partial J(w)}{\partial w_{01}} \\ \frac{\partial J(w)}{\partial w_{02}} \\ \frac{\partial J(w)}{\partial w_{12}} \end{bmatrix}$$

Each partial derivative is evaluated at w^{old}.

Example: Computing Partial Derivatives

$$\begin{aligned} \mathrm{J(w)} = \; & [b_{x0} + w_{01}(r_{x1} - b_{x1}) + w_{02}(r_{x2} - b_{x2}) \; - r_{x0}]^2 \\ & + \; [b_{x1} + w_{01}(r_{x0} - b_{x0}) + w_{12}(r_{x2} - b_{x2}) \; - r_{x1}]^2 \\ & + \; [b_{x2} + w_{02}(r_{x0} - b_{x0}) + w_{12}(r_{x1} - b_{x1}) \; - r_{x2}]^2 \end{aligned}$$

Reminder: $\frac{\partial((\mathrm{ax+b})^2)}{\partial \mathrm{x}} = 2\;(\mathrm{ax} + \mathrm{b})\;\mathrm{a}$

$$\begin{aligned} \frac{\partial J(w)}{\partial w_{01}} = \; & 2[b_{x0} + w_{01}(r_{x1} - b_{x1}) + w_{02}(r_{x2} - b_{x2}) \; - r_{x0}]\,(r_{x1} - b_{x1}) \\ & +2\;[b_{x1} + w_{01}(r_{x0} - b_{x0}) + w_{12}(r_{x2} - b_{x2}) \; - r_{x1}]\,(r_{x0} - b_{x0}) \end{aligned}$$

Evaluate each partial derivative at w^{old} to compute the gradient direction.

Interpolation Weights

- **We have the optimization problem, now what?**

$$J(w) = \sum_{x}\left(\left[b_{xi} + \sum_{j \in N(i;x)} w_{ij}\left(r_{xj} - b_{xj}\right)\right] - r_{xi}\right)^2$$

- **Gradient descent:**
  - **Iterate until convergence:** $\boldsymbol{w} \leftarrow \boldsymbol{w} - \eta \boldsymbol{\nabla_w J}$ $\quad \eta$ … learning rate
  - **where $\nabla_w J$ is the gradient (derivative evaluated on data):**

$$\nabla_w J = \left[\frac{\partial J(w)}{\partial w_{ij}}\right] = 2\sum_{x,i}\left(\left[b_{xi} + \sum_{k \in N(i;x)} w_{ik}\left(r_{xk} - b_{xk}\right)\right] - r_{xi}\right)\left(r_{xj} - b_{xj}\right)$$

for $\boldsymbol{j \in \{N(i;x), \forall i, \forall x\}}$

else $\dfrac{\partial J(w)}{\partial w_{ij}} = \mathbf{0}$

  - **Note:** We fix movie ***i***, go over all $\boldsymbol{r_{xi}}$, for every movie $\boldsymbol{j} \in \boldsymbol{N(i;x)}$, we compute $\dfrac{\boldsymbol{\partial J(w)}}{\boldsymbol{\partial w_{ij}}}$

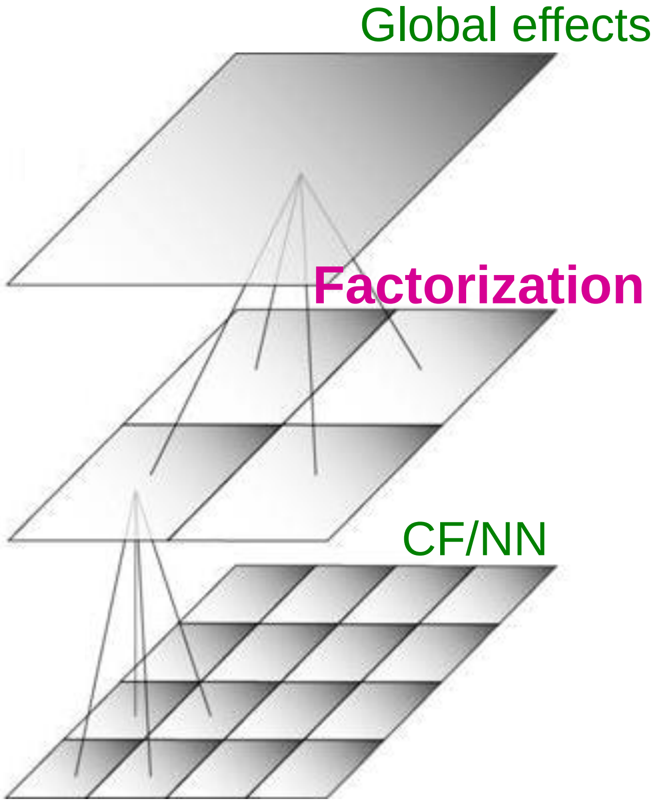
while $|w_{new} - w_{old}| > \varepsilon$**:**

$w_{old} = w_{new}$

$w_{new} = w_{old} - \eta \cdot \nabla w_{old}$

Interpolation Weights

- **So far:** $\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij}(r_{xj} - b_{xj})$
  - Weights $\boldsymbol{w_{ij}}$ derived based on their role; **no use of an arbitrary similarity measure** ($\boldsymbol{w_{ij}} \neq \boldsymbol{s_{ij}}$)
  - Explicitly account for interrelationships among the neighboring movies
- **Next: Latent factor model**
  - Extract "regional" correlations



Performance of Various Methods

Global average: 1.1296

User average: 1.0651

Movie average: 1.0533

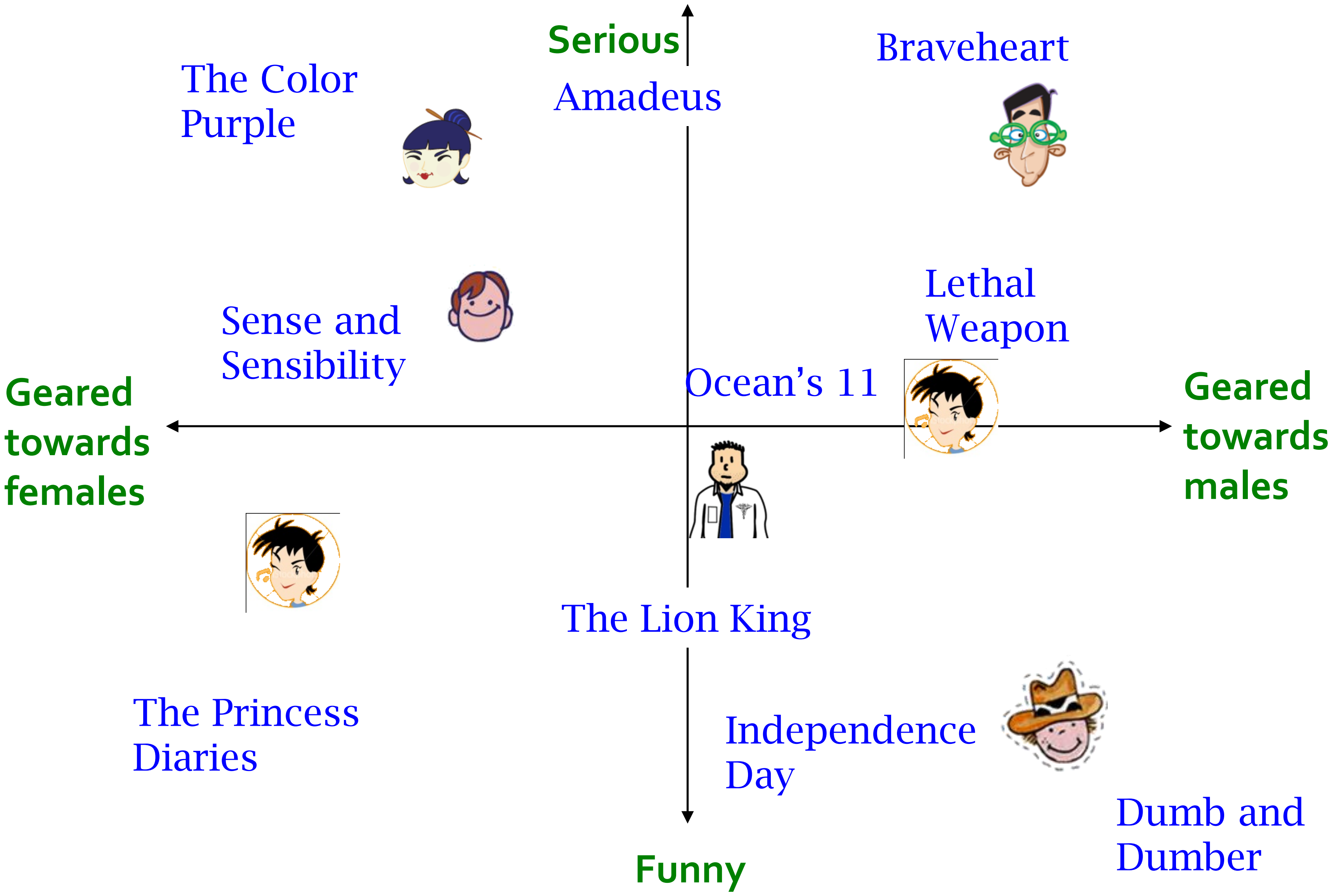
Netflix: 0.9514

Basic Collaborative filtering: 0.94

CF+Biases+learned weights: 0.91

Grand Prize: 0.8563

Latent Factor Models (e.g., SVD)



Serious

The Color Purple

Amadeus

Braveheart

Sense and Sensibility

Lethal Weapon

Ocean’s 11

Geared towards females

Geared towards males

The Lion King

The Princess Diaries

Independence Day

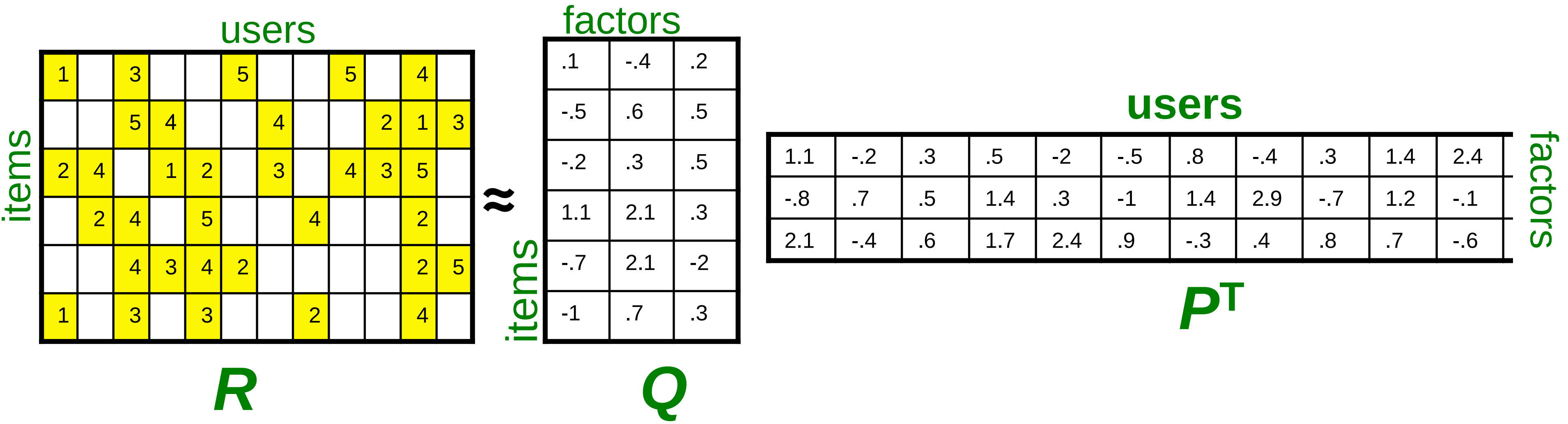
Dumb and Dumber

Funny

Latent Factor Models

SVD: $A = U \Sigma V^T$

- “SVD” on Netflix data: $\mathbf{R} \approx \boldsymbol{Q} \cdot \boldsymbol{P}^T$



1		3			5			5		4	
		5	4			4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6

- For now let’s assume we can approximate the rating matrix $\boldsymbol{R}$ as a product of “thin” $\boldsymbol{Q} \cdot \boldsymbol{P}^T$
  - $\boldsymbol{R}$ has missing entries but let’s ignore that for now!
    - Basically, we will want the reconstruction error to be small on known ratings and we don’t care about the values on the missing ones

Ratings as Products of Factors

- **How to estimate the missing rating of user *x* for item *i*?**

$$\hat{r}_{xi} = q_i \cdot p_x$$

$$= \sum_f q_{if} \cdot p_{xf}$$

q_i = row i of Q

p_x = column x of P^T

users

items

1		3			5			5		4	
		5	4	?		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

≈

items

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

factors

Q

•

users

factors

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

P^T

Ratings as Products of Factors

- How to estimate the missing rating of user *x* for item *i*?

users

items

1		3			5			5		4	
		5	4	**?**		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

$\approx$

$$\hat{r}_{xi} = q_i \cdot p_x$$

$$= \sum_f q_{if} \cdot p_{xf}$$

q_i = row *i* of ***Q***
p_x = column *x* of P^T

items

.1	-.4	.2
-.5	**.6**	**.5**
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

factors

Q

•

users

factors

1.1	-.2	.3	.5	**-2**	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	**.3**	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	**2.4**	.9	-.3	.4	.8	.7	-.6	.1

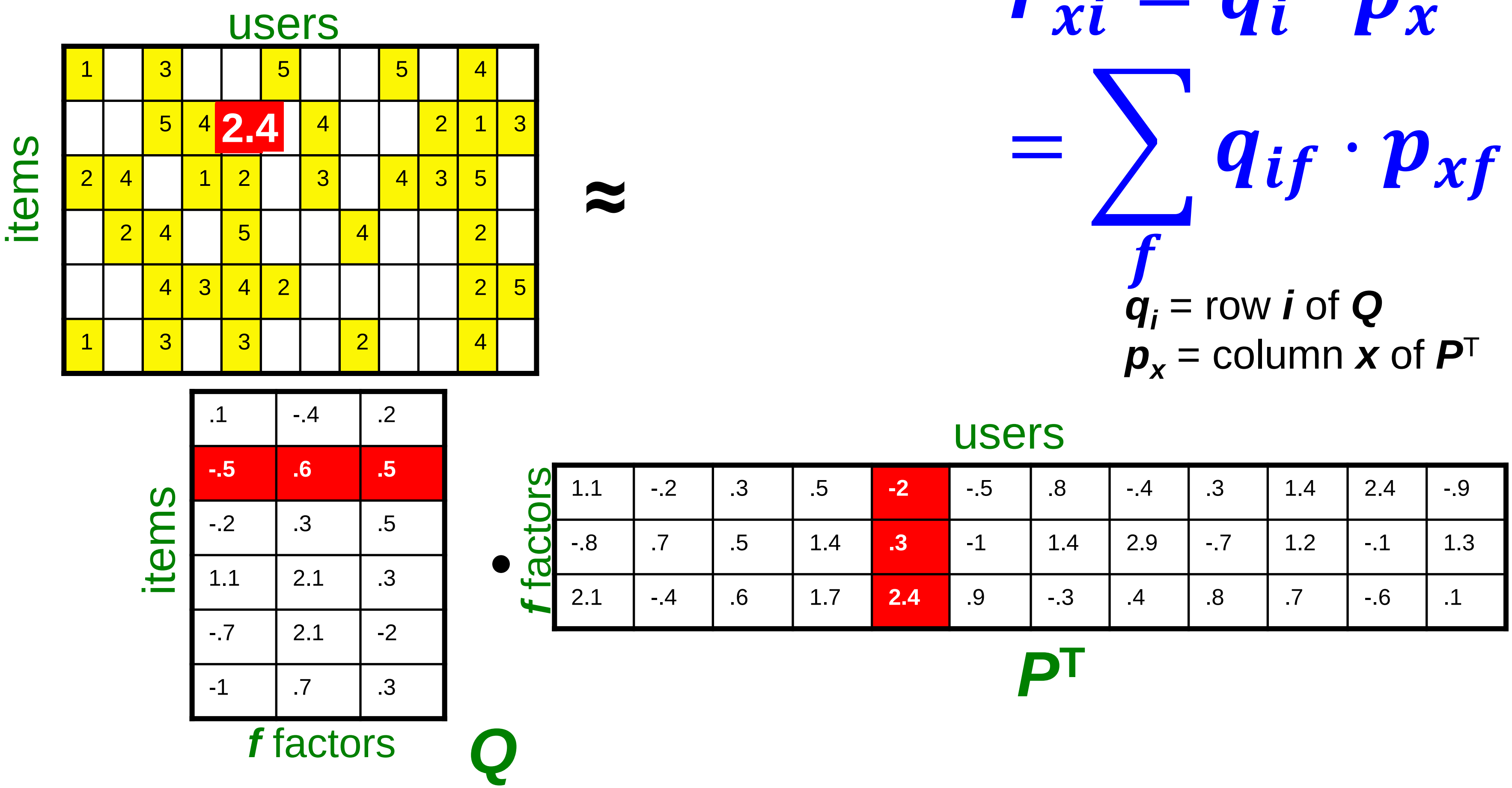
P^T

Ratings as Products of Factors

- **How to estimate the missing rating of user *x* for item *i*?**

$$\hat{r}_{xi} = q_i \cdot p_x$$
$$= \sum_f q_{if} \cdot p_{xf}$$

q_i = row i of Q
p_x = column x of P^T



1		3			5			5		4	
		5	4	2.4		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

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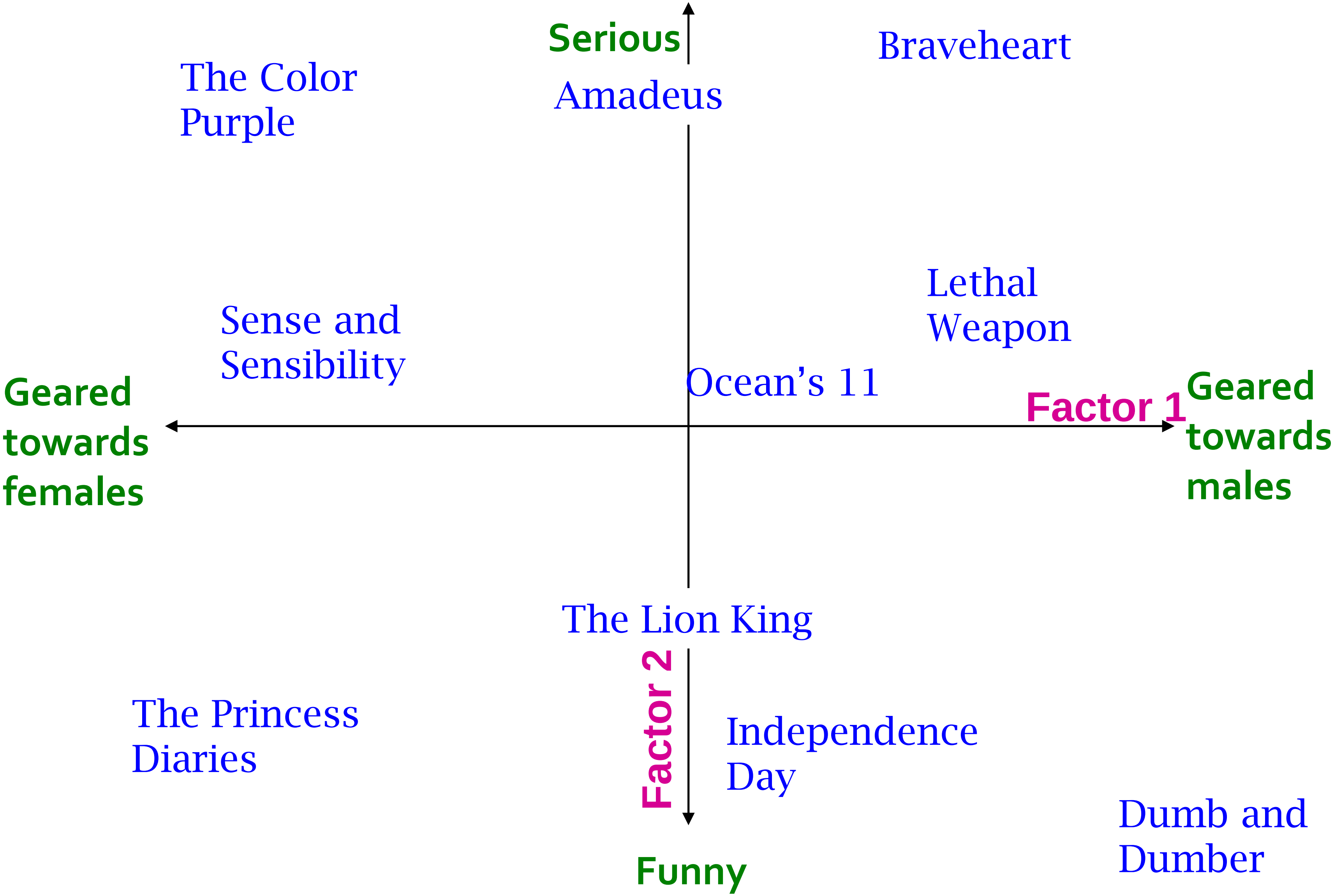
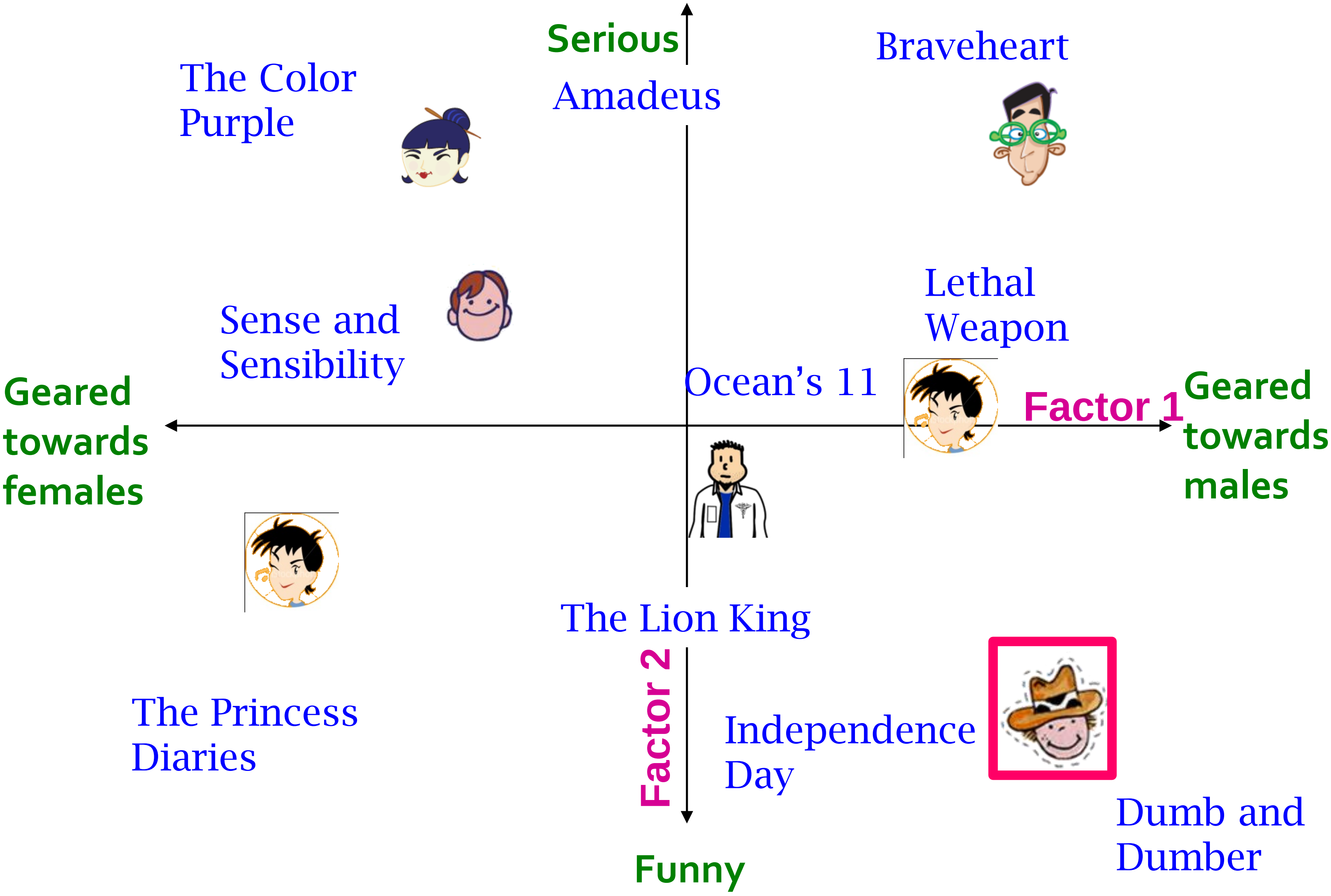
Latent Factor Models



Latent Factor Models



Serious

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Sense and Sensibility

Ocean's 11

Factor 1

Geared towards females

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The Lion King

Factor 2

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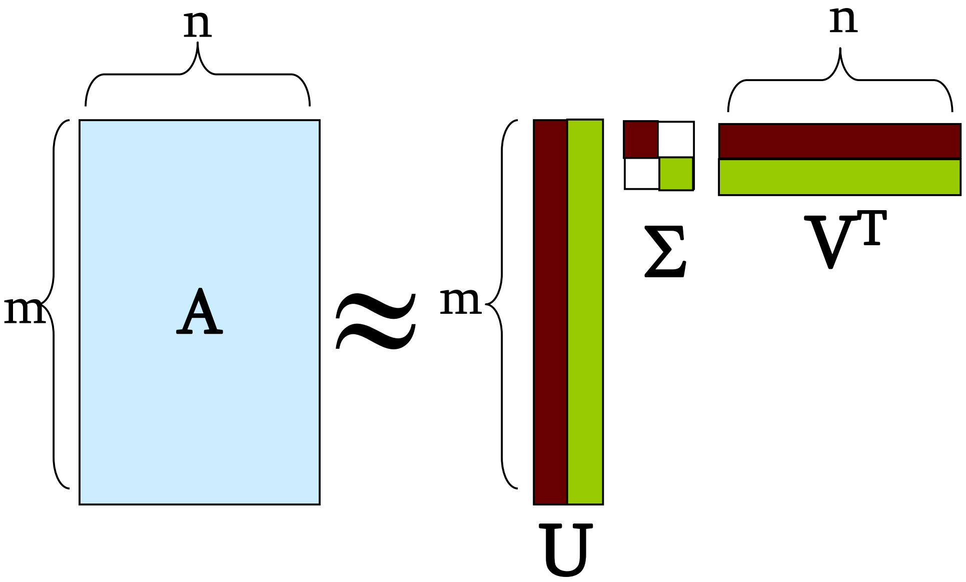
Independence Day

Dumb and Dumber

Funny

FYI: Singular Value Decomposition (SVD)

- **FYI, SVD:**
  - **A**: Input data matrix
  - **U**: Left singular vecs
  - **V**: Right singular vecs
  - Σ: Singular values



- **So in our case:**
  "SVD" on Netflix data: $R \approx Q \cdot P^T$
  $A = R, \; Q = U, \; P^T = \Sigma V^T$

$$\hat{r}_{xi} = q_i \cdot p_x$$

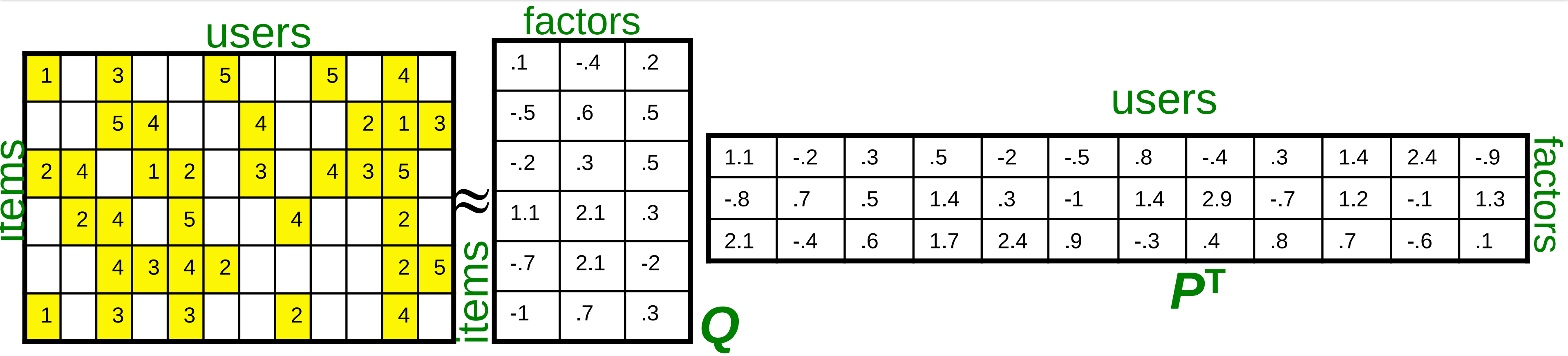
SVD: More good stuff

- **We already know that SVD gives minimum reconstruction error (Sum of Squared Errors):**

$$\min_{U,\mathrm{V},\Sigma} \sum_{ij \in A} \left(A_{ij} - \left[U\Sigma V^{\mathrm{T}}\right]_{ij}\right)^2$$

- **Note two things:**
  - **SSE** and **RMSE** are monotonically related:
    - $\boldsymbol{RMSE = \frac{1}{c}\sqrt{SSE}}$ **Great news: SVD is minimizing RMSE**
  - **Complication:** The sum in SVD error term is over all entries (no-rating in interpreted as zero-rating). But our ***R*** has missing entries!

Latent Factor Models



1		3			5			5		4	
		5	4			4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

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-.5	.6	.5
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1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

- **SVD isn't defined when entries are missing!**
- **Use specialized methods to find *P, Q***
  - $\min_{P,Q} \sum_{(i,x)\in \mathrm{R}} \left(r_{xi} - q_i \cdot p_x\right)^2$ $\qquad \hat{r}_{xi} = q_i \cdot p_x$
  - **Note:**
    - We don't require cols of ***P, Q*** to be orthogonal/unit length
    - ***P, Q*** map users/movies to a latent space
    - The most popular model among Netflix contestants

Finding the Latent Factors

General Concept: Overfitting

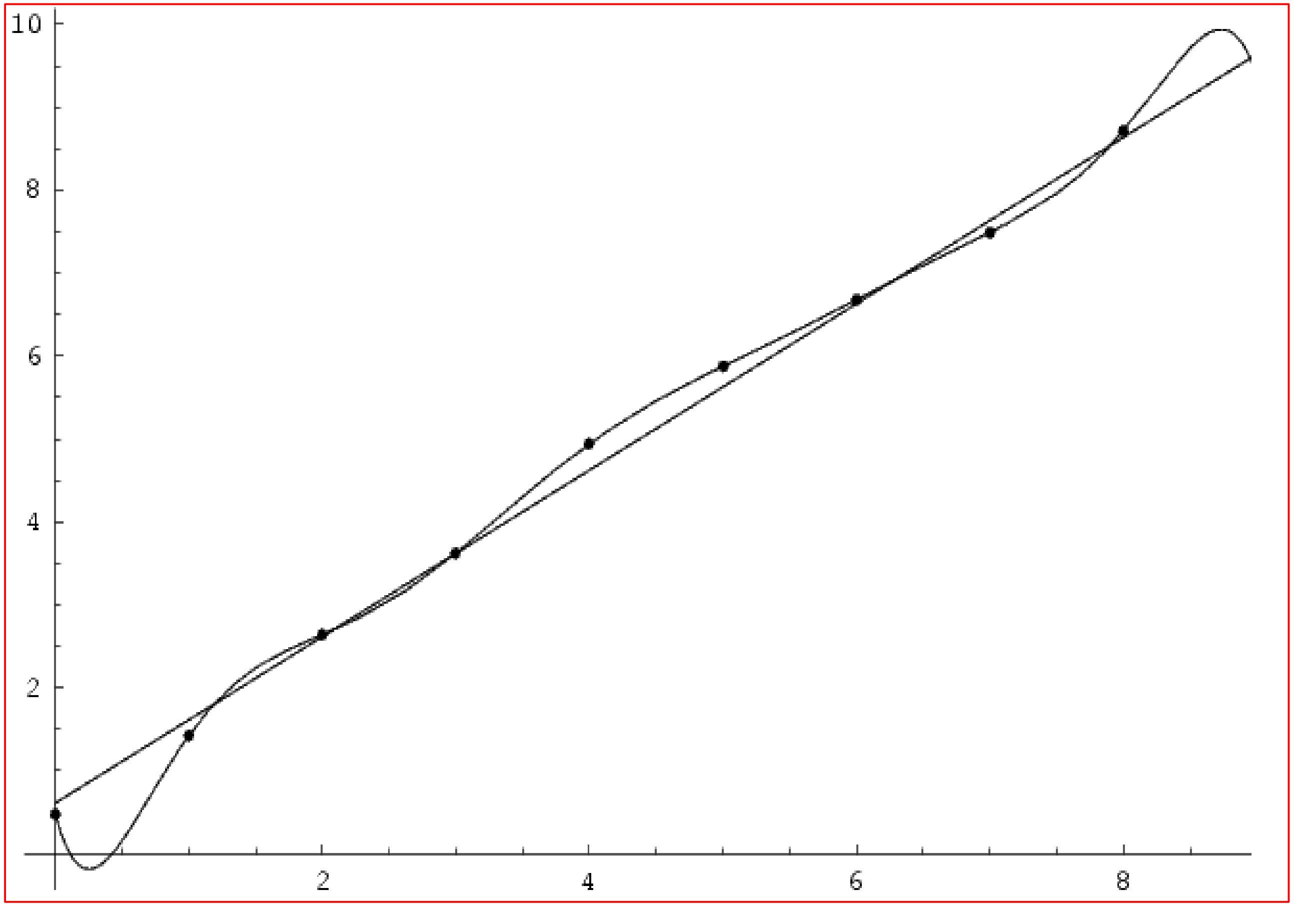


Image source: Wikipedia

Almost-linear data is fit to a linear function and a polynomial function.

Polynomial model fits perfectly to data.

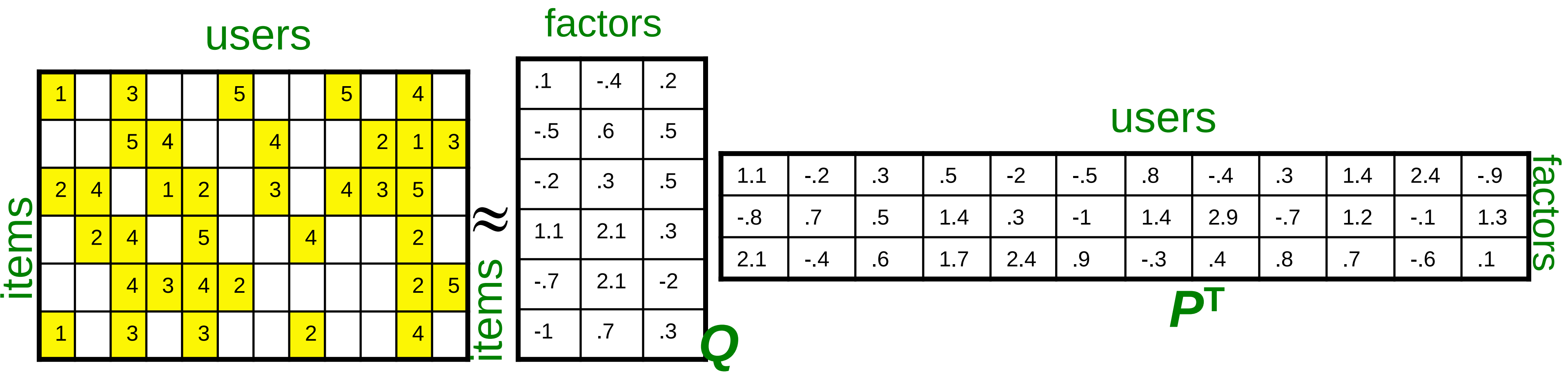
Linear model has some error in the training set.

Linear model is expected to perform better on test data, because it filters out noise.

Latent Factor Models

- **Our goal is to find P and Q such that:**

$$\min_{P,Q} \sum_{(i,x)\in R} \left(r_{xi} - q_i \cdot p_x\right)^2$$



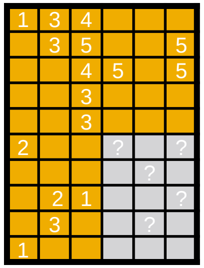
1		3			5			5		4	
		5	4			4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

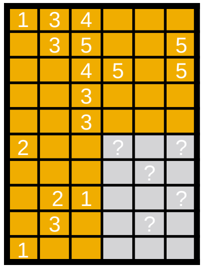
Back to Our Problem

- Want to minimize SSE for unseen test data
- **Idea:** Minimize SSE on training data
  - Want large ***k*** (# of factors) to capture all the signals
  - But, **SSE** on test data begins to rise for ***k*** > 2
- This is a classical example of **overfitting:**
  - With too much freedom (too many free parameters) the model starts fitting noise
    - That is it fits too well the training data and thus **not generalizing** well to unseen test data



1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			?		?
				?	
	2	1			?
	3			?	
1					

Dealing with Missing Entries



1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			?		?
				?	
	2	1			?
	3			?	
1					

- **To solve overfitting we introduce regularization:**
  - Allow rich model where there are sufficient data
  - Shrink aggressively where data are scarce

$$\min_{P,Q} \underbrace{\sum_{training} (r_{xi} - q_i p_x)^2}_{\text{“error”}} + \left[\underbrace{\lambda_1 \sum_x \|p_x\|^2 + \lambda_2 \sum_i \|q_i\|^2}_{\text{“length”}} \right]$$

λ_1, λ_2 … user set regularization parameters

Note: We do not care about the “raw” value of the objective function, but we care in P,Q that achieve the minimum of the objective

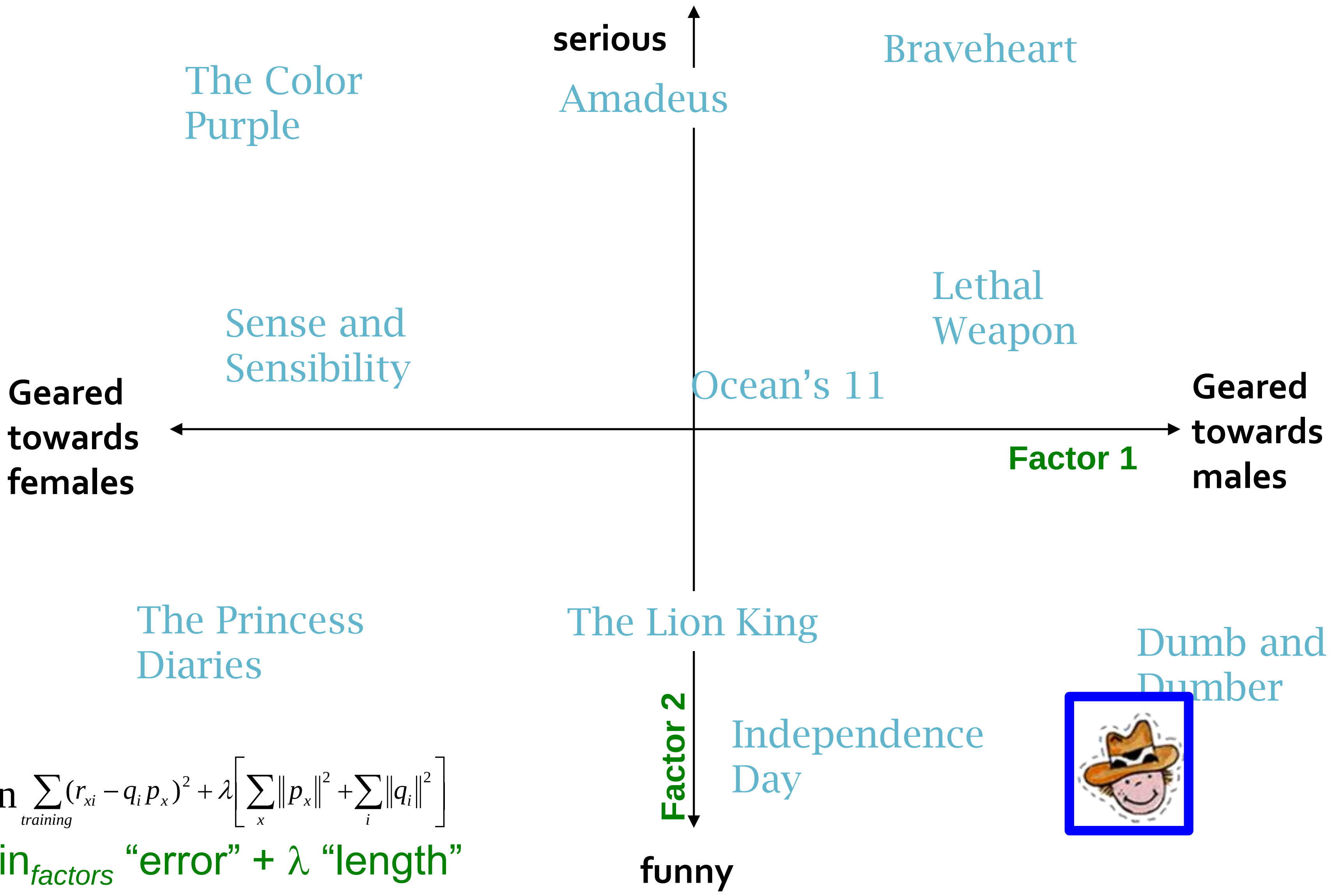
Regularization

$$\min_{P,Q} \sum_{training} \underbrace{(r_{xi} - q_i p_x)^2}_{\text{“error”}} + \left[\lambda_1 \underbrace{\sum_x \left\| p_x \right\|^2 + \lambda_2 \sum_i \left\| q_i \right\|^2}_{\text{“length”}} \right]$$

λ_1, λ_2 … user set regularization parameters

- What happens if the user x has rated hundreds of movies?
  - The error term will dominate, and we’ll get a rich model
  - Noise is less of an issue because we have lots of data
- What happens if the user x has rated only a few movies?
  - Length term for p_x will have more effect, and we’ll get a simple model
- Same argument applies for items

The Effect of Regularization



serious

funny

Geared towards females

Geared towards males

Factor 1

Factor 2

The Color Purple

Amadeus

Braveheart

Lethal Weapon

Sense and Sensibility

Ocean's 11

The Princess Diaries

The Lion King

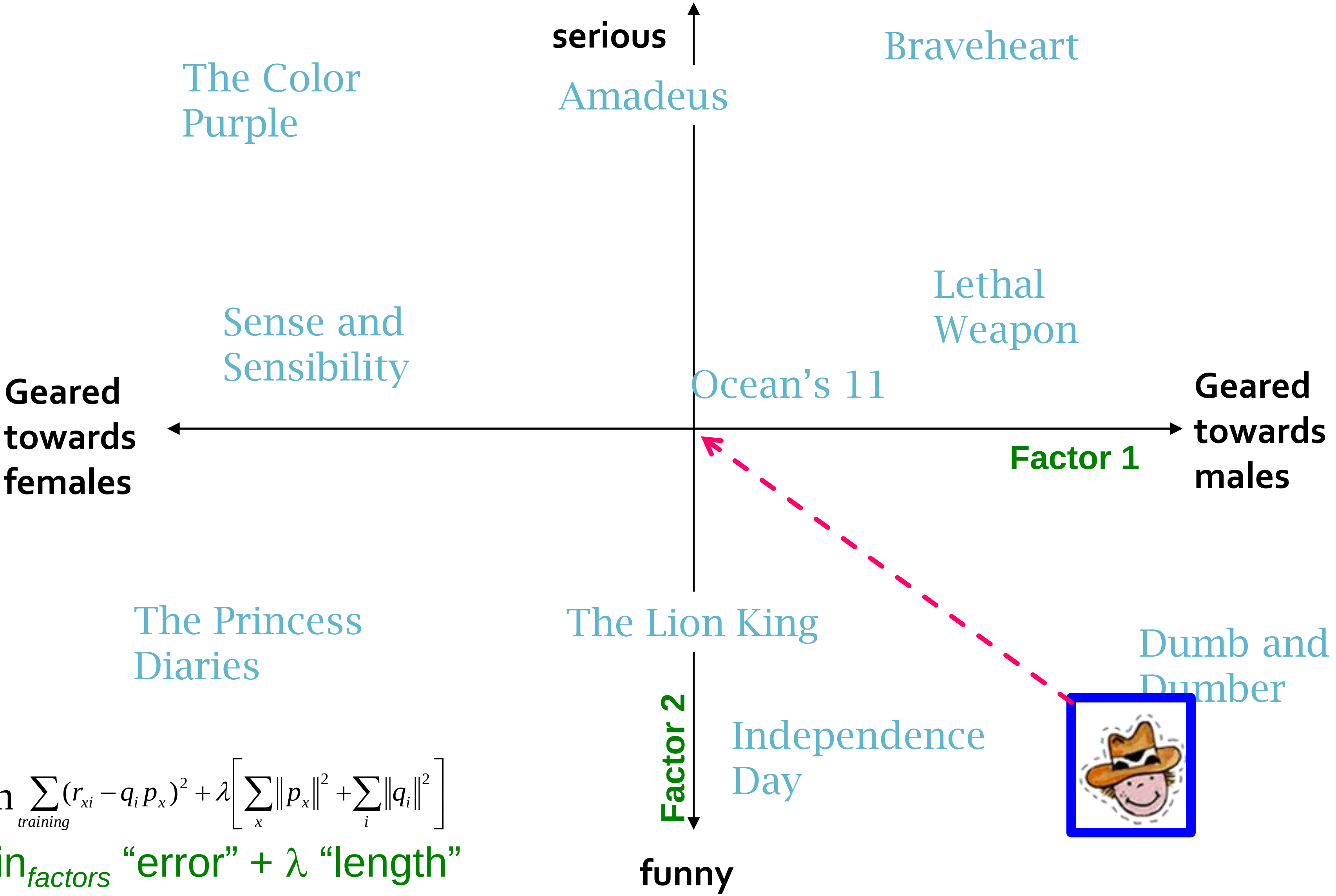
Independence Day

Dumb and Dumber

$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \lambda \left[\sum_x \|p_x\|^2 + \sum_i \|q_i\|^2 \right]$$

$\min_{factors}$ “error” + λ “length”

The Effect of Regularization



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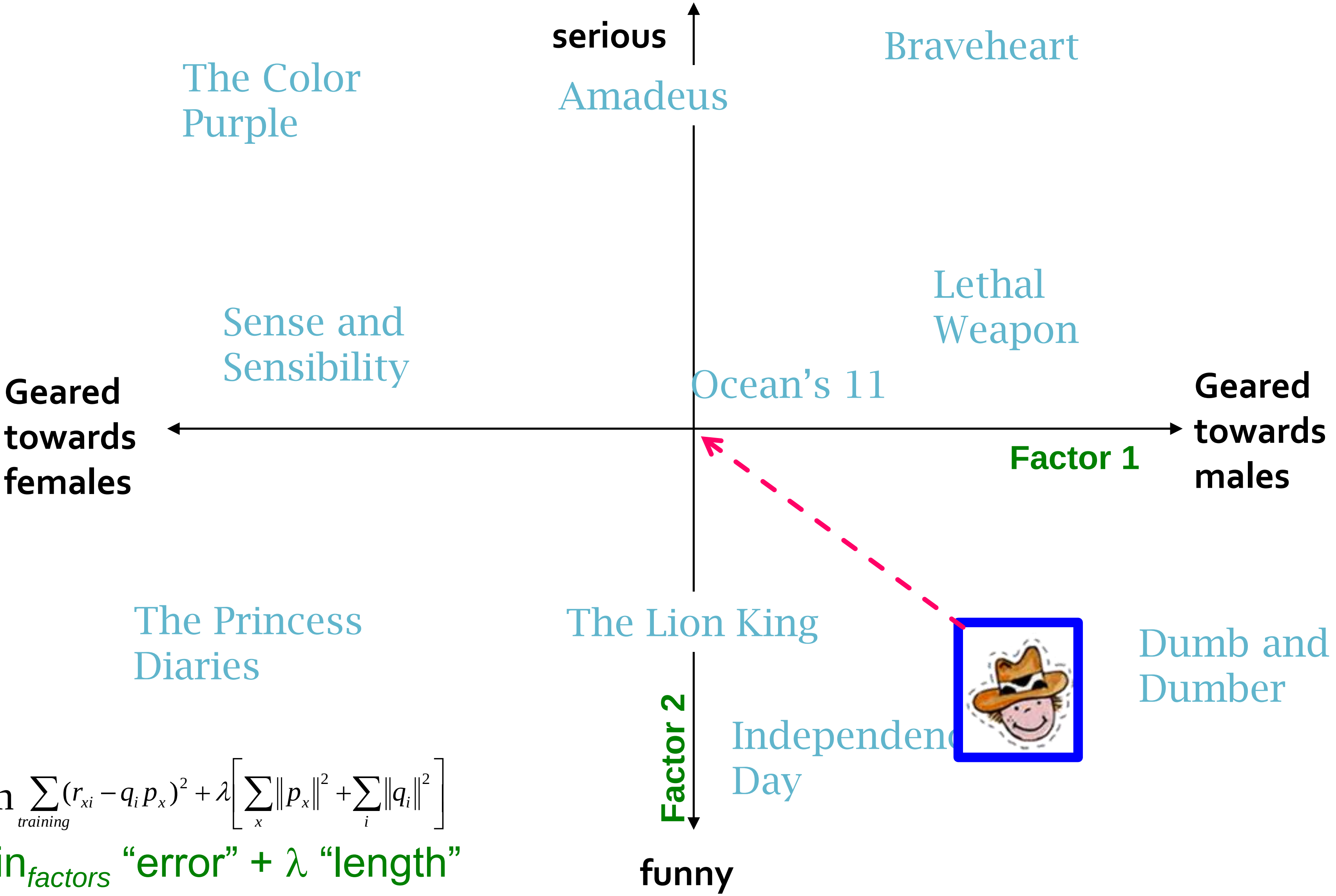

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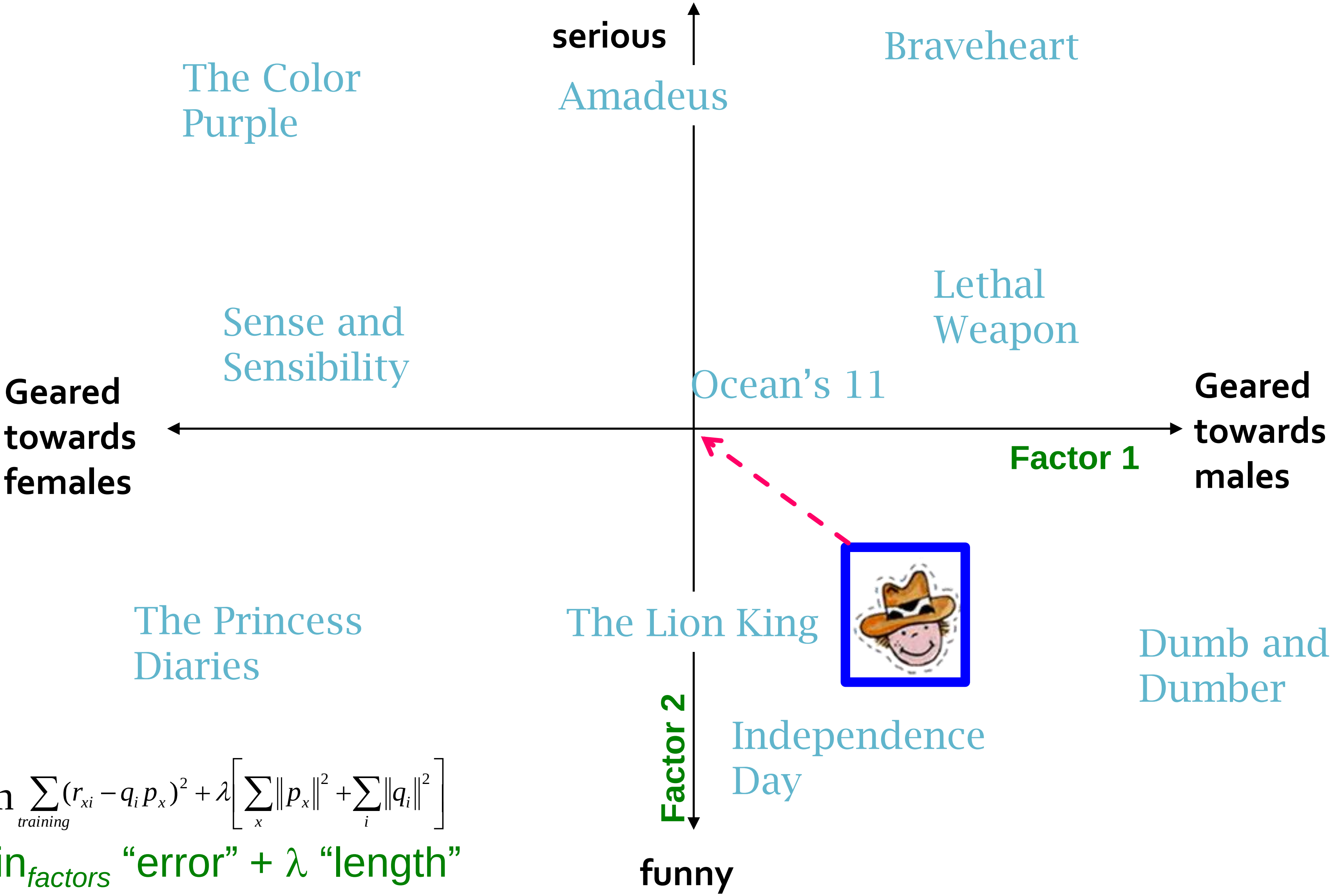

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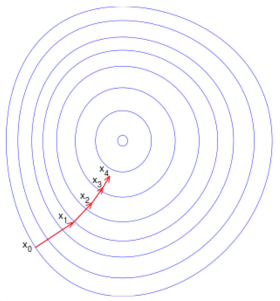
$\min_{factors}$ "error" + λ "length"

Gradient Descent



- **Want to find matrices *P* and *Q*:**

$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \left[\lambda_1 \sum_x \|p_x\|^2 + \lambda_2 \sum_i \|q_i\|^2 \right]$$

- **Gradient descent:**
  - Initialize ***P*** and ***Q*** (using SVD, pretend missing ratings are 0)
  - Do gradient descent:
    - $\boldsymbol{P \leftarrow P - \eta \cdot \nabla P}$
    - $\boldsymbol{Q \leftarrow Q - \eta \cdot \nabla Q}$
    - where ***∇Q*** is gradient/derivative of matrix ***Q***:
    $\nabla Q = [\nabla q_{if}]$ and $\nabla q_{if} = \sum_{x,i} -2(r_{xi} - q_i p_x) p_{xf} + 2\lambda_2 q_{if}$
      - Here $\boldsymbol{q_{if}}$ is entry ***f*** of row $\boldsymbol{q_i}$ of matrix ***Q***
  - **Observation: Computing gradients is slow!**

How to compute gradient of a matrix?
Compute gradient of every element independently!

Example

$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \left[\lambda_1 \sum_x \|p_x\|^2 + \lambda_2 \sum_i \|q_i\|^2 \right]$$

Assume we want 3 factors per user and item: $p_x = \begin{bmatrix} p_{x0} \\ p_{x1} \\ p_{x2} \end{bmatrix}$ $q_i = \begin{bmatrix} q_{i0} \\ q_{i1} \\ q_{i2} \end{bmatrix}$

Rewrite objective as:

$$\min \sum_{x,i} [r_{xi} - (q_{i0}p_{x0} + q_{i1}p_{x1} + q_{i2}p_{x2})]^2$$

$$+\lambda_1 \sum_x \left(p_{x0}^2 + p_{x1}^2 + p_{x2}^2\right)$$

$$+\lambda_2 \sum_i \left(q_{i0}^2 + q_{i1}^2 + q_{i2}^2\right)$$

Example

$$\min \sum_{x,i} [r_{xi} - (q_{i0}p_{x0} + q_{i1}p_{x1} + q_{i2}p_{x2})]^2$$
$$+\lambda_1 \sum_{x} (p_{x0}^2 + p_{x1}^2 + p_{x2}^2)$$
$$+\lambda_2 \sum_{i} (q_{i0}^2 + q_{i1}^2 + q_{i2}^2)$$

$$p_x = \begin{bmatrix} p_{x0} \\ p_{x1} \\ p_{x2} \end{bmatrix} \qquad q_i = \begin{bmatrix} q_{i0} \\ q_{i1} \\ q_{i2} \end{bmatrix}$$

Compute gradient for variable q_{i0}:

$$\nabla q_{i0} = \sum_{x,i} -2(r_{xi} - (q_{i0}p_{x0} + q_{i1}p_{x1} + q_{i2}p_{x2}))p_{x0} + 2\lambda_2 q_{i0}$$

Do the same for every free variable

Gradient Descent - Computation Cost

$\nabla Q = [\nabla q_{if}]$ and $\nabla q_{if} = \sum_{x,i} -2\big(r_{xi} - q_i p_x\big) p_{xf} + 2\lambda_2 q_{if}$

- How many free variables do we have?

  (# of users + # of items) . (# of factors)

- Which ratings do we process to compute ∇q_{if} ?

  All ratings for item i

- Which ratings do we process to compute ∇p_{xf} ?

  All ratings for user x

- What is the complexity of one iteration?

  O(# of ratings . # of factors)

Stochastic Gradient Descent

- *Gradient Descent (GD)*: Update all free variables in one step. Need to process all ratings.
- *Stochastic Gradient Descent (SGD):* Update the free variables associated with a single rating in one step.
  - Need many more steps to converge
  - Each step is much faster
  - In practice: SGD much faster than GD

- **GD:** $\boldsymbol{Q} \leftarrow \boldsymbol{Q} - \eta\left[\sum_{\boldsymbol{r_{xi}}} \nabla \boldsymbol{Q}(\boldsymbol{r_{xi}})\right]$
- **SGD:** $\boldsymbol{Q} \leftarrow \boldsymbol{Q} - \mu \nabla \boldsymbol{Q}(\boldsymbol{r_{xi}})$

Stochastic Gradient Descent

$$\nabla q_{if} = \sum_{x,i} -2(r_{xi} - q_i p_x \,) p_{xf} + 2\lambda_2 q_{if}$$

$$\nabla p_{xf} = \sum_{x,i} -2(r_{xi} - q_i p_x \,) q_{if} + 2\lambda_1 p_{xf}$$

Which free variables are associated with rating r_{xi}?

$$p_x = \begin{bmatrix} p_{x0} \\ p_{x1} \\ . \\ . \\ p_{xk} \end{bmatrix} \qquad q_i = \begin{bmatrix} q_{i0} \\ q_{i1} \\ . \\ . \\ q_{ik} \end{bmatrix}$$

Stochastic Gradient Descent

$$\nabla q_{if} = \sum_{x,i} -2(r_{xi} - q_i p_x\ )p_{xf} + 2\lambda_2 q_{if}$$

$$\nabla p_{xf} = \sum_{x,i} -2(r_{xi} - q_i p_x\ )q_{if} + 2\lambda_1 p_{xf}$$

For each *r_{xi}*:

$\varepsilon_{xi} = (r_{xi} - q_i \cdot p_x\ )$ (derivative of the "error")

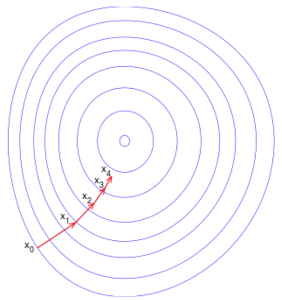
$q_i \leftarrow q_i + \mu_1\ (\varepsilon_{xi}\ p_x - \lambda_2\ q_i)$ (update equation)

$p_x \leftarrow p_x + \mu_2\ (\varepsilon_{xi}\ q_i - \lambda_1\ p_x)$ (update equation)

μ … learning rate

Note: The operations above are vector operations

Stochastic Gradient Descent



- **Stochastic gradient decent:**
  - Initialize ***P*** and ***Q*** (using SVD, pretend missing ratings are 0)
  - Then iterate over the ratings (multiple times if necessary) and update factors:

  For each *r*$_{xi}$**:**

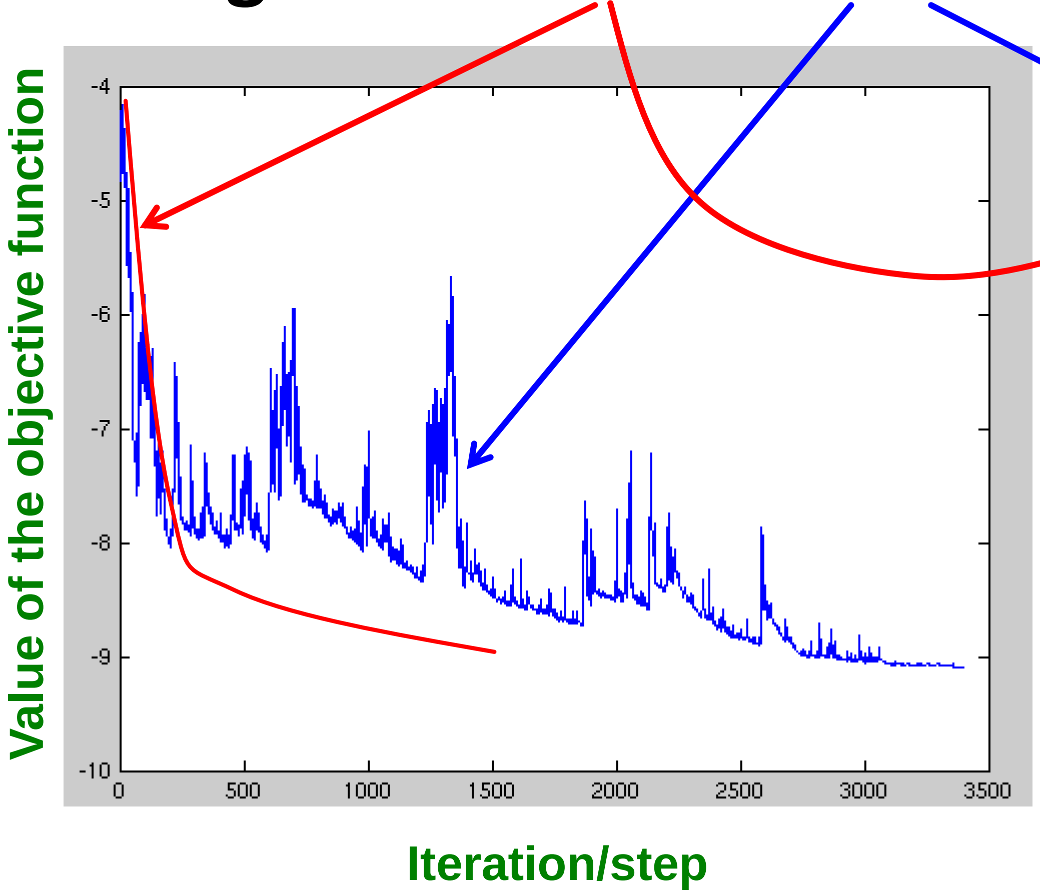
    - $\varepsilon_{xi} = (r_{xi} - q_i \cdot p_x)$ (derivative of the "error")
    - $q_i \leftarrow q_i + \mu_1 (\varepsilon_{xi}\, p_x - \lambda_2\, q_i)$ (update equation)
    - $p_x \leftarrow p_x + \mu_2 (\varepsilon_{xi}\, q_i - \lambda_1\, p_x)$ (update equation)

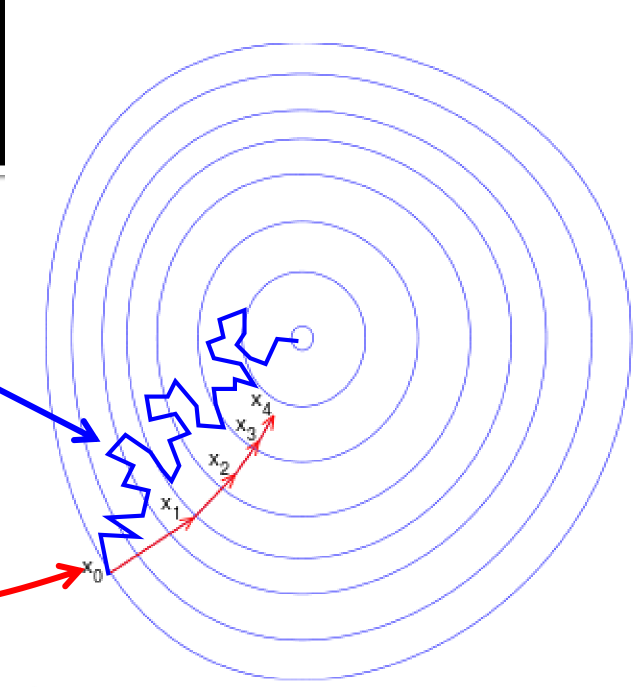
  μ … learning rate

- **2 for loops:**
  - For until convergence:
    - For each **r**$_{xi}$
      - Compute gradient, do a "step"

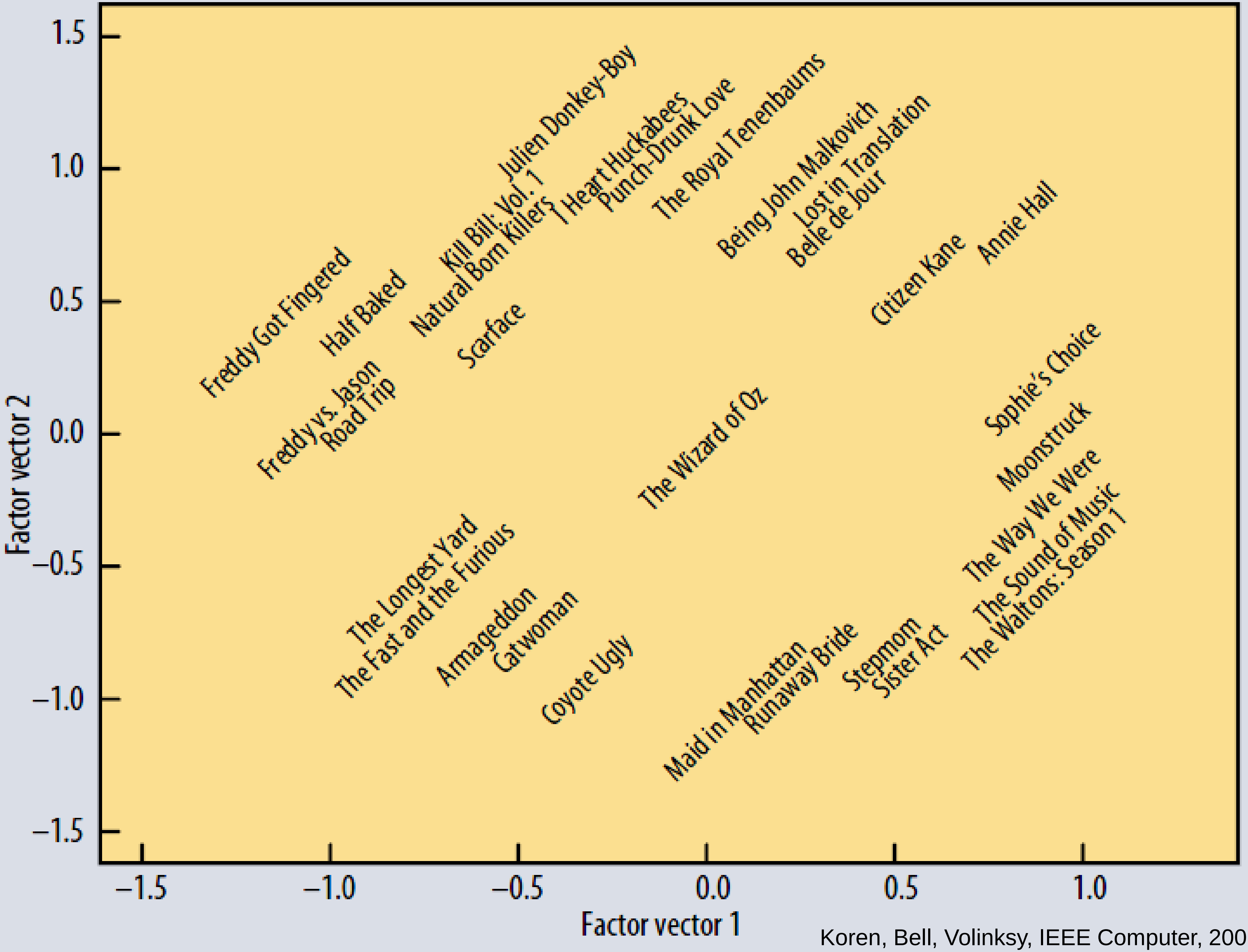
SGD vs. GD

- **Convergence of GD vs. SGD**





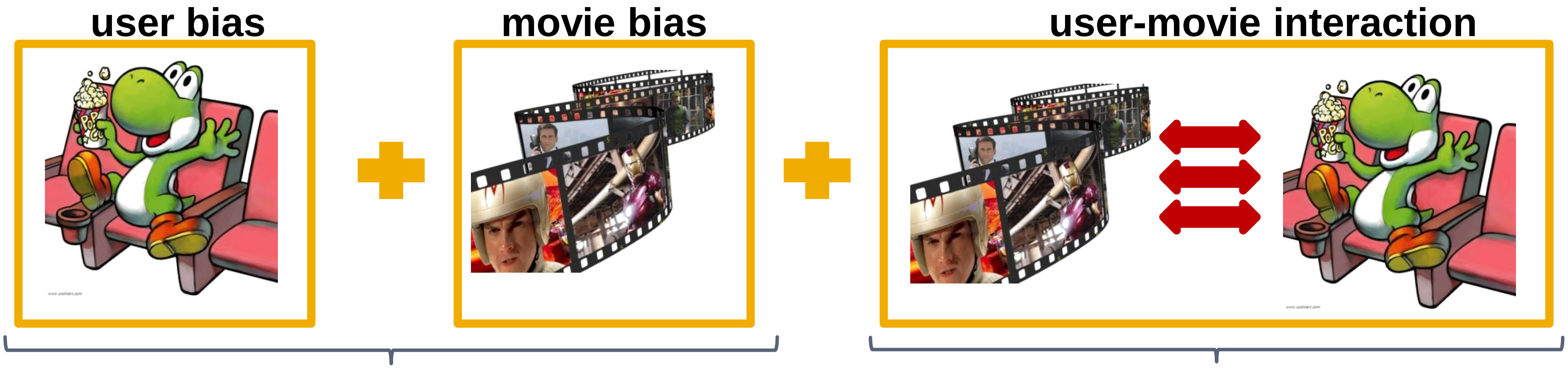
GD improves the value of the objective function at every step.
SGD improves the value but in a “noisy” way.
GD takes fewer steps to converge but each step takes much longer to compute.
In practice, **SGD** is much faster!



Koren, Bell, Volinksy, IEEE Computer, 2009

Extending Latent Factor Model to Include Biases

Modeling Biases and Interactions



Baseline predictor

- Separates users and movies
- Benefits from insights into user’s behavior
- Among the main practical contributions of the competition

User-Movie interaction

- Characterizes the matching between users and movies
- Attracts most research in the field
- Benefits from algorithmic and mathematical innovations

- $\boldsymbol{\mu}$ = overall mean rating
- $\boldsymbol{b_x}$ = bias of user $\boldsymbol{x}$
- $\boldsymbol{b_i}$ = bias of movie $\boldsymbol{i}$

Baseline Predictor

- We have expectations on the rating by user ***x*** of movie ***i***, even without estimating ***x***'s attitude towards movies like ***i***

➕

- Rating scale of user ***x***
- Values of other ratings user gave recently (day-specific mood, anchoring, multi-user accounts)

- (Recent) popularity of movie ***i***
- Selection bias; related to number of ratings user gave on the same day ("frequency")

Putting It All Together

$$r_{xi} = \underbrace{\mu}_{\text{Overall mean rating}} + \underbrace{b_x}_{\text{Bias for user } x} + \underbrace{b_i}_{\text{Bias for movie } i} + \underbrace{q_i \cdot p_x}_{\text{User-Movie interaction}}$$

- **Example:**
  - Mean rating: ***μ = 3.7***
  - You are a critical reviewer: your ratings are 1 star lower than the mean: ***b_x = -1***
  - Star Wars gets a mean rating of *0.5* higher than average movie: ***b_i = + 0.5***
  - Predicted rating for you on Star Wars:
    = 3.7 - 1 + 0.5 = 3.2

Fitting the New Model

- **Solve:**

$$\min_{Q,P} \sum_{(x,i)\in R} \Big(r_{xi} - (\mu + b_x + b_i + q_i\, p_x)\Big)^2$$

goodness of fit

$$+ \left(\lambda_1 \sum_i \|q_i\|^2 + \lambda_2 \sum_x \|p_x\|^2 + \lambda_3 \sum_x \|b_x\|^2 + \lambda_4 \sum_i \|b_i\|^2 \right)$$

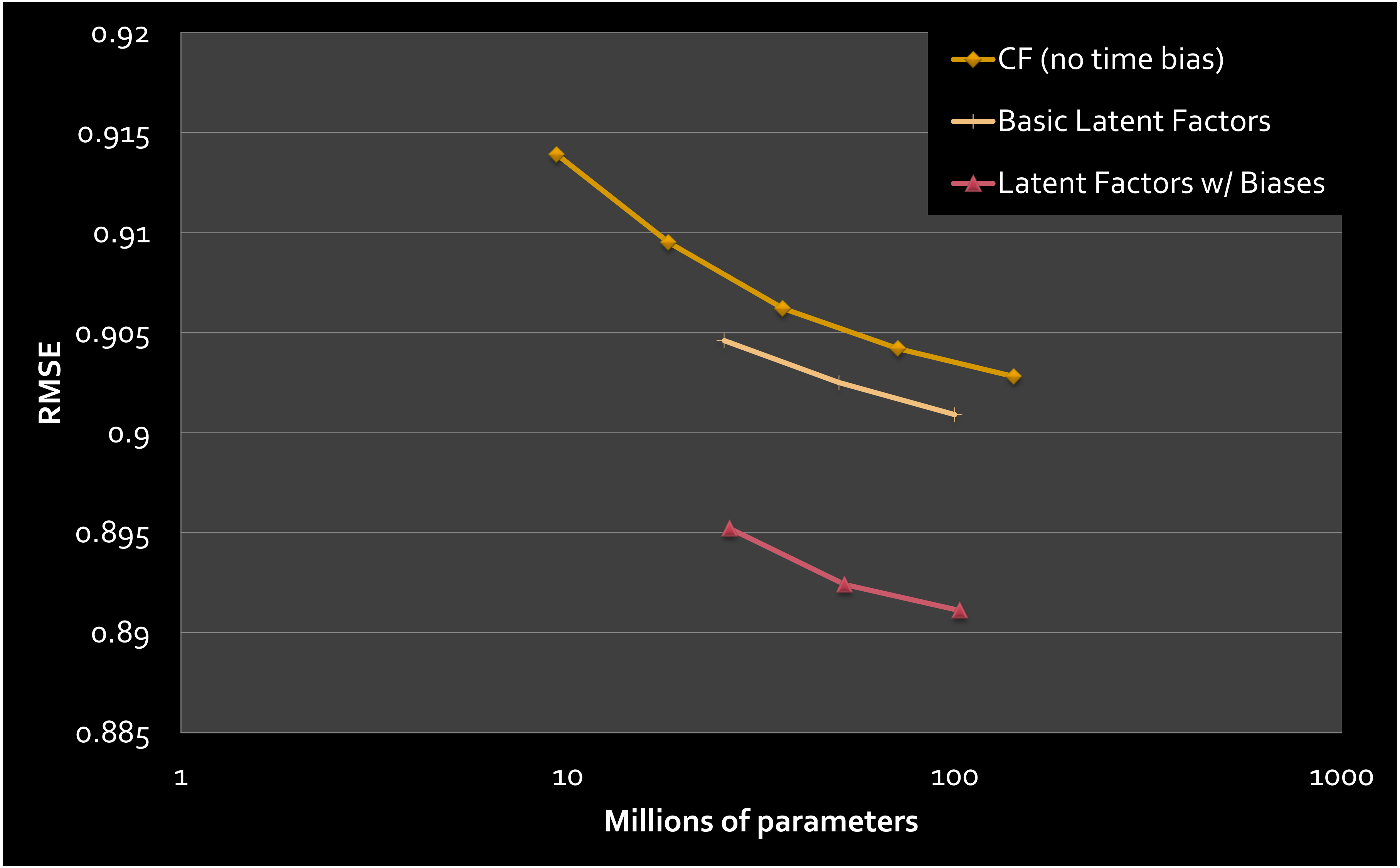
↑

λ is selected via grid-search on a validation set

regularization

- **Stochastic gradient decent to find parameters**
  - **Note:** Both biases ***b_x*, *b_i*** as well as interactions ***q_i*, *p_x*** are treated as parameters (we estimate them)

Performance of Various Methods



Performance of Various Methods

Global average: 1.1296

User average: 1.0651

Movie average: 1.0533

Netflix: 0.9514

Basic Collaborative filtering: 0.94

Collaborative filtering++: 0.91

Latent factors: 0.90

Latent factors+Biases: 0.89

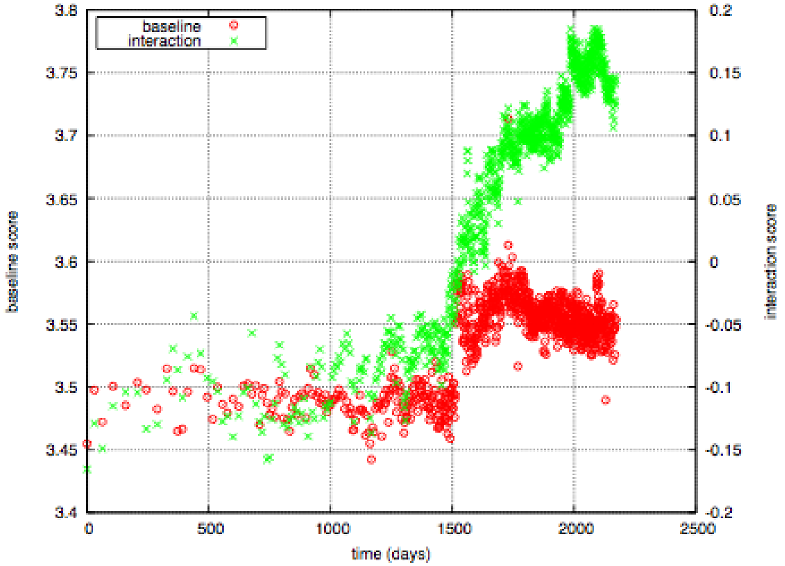
Grand Prize: 0.8563

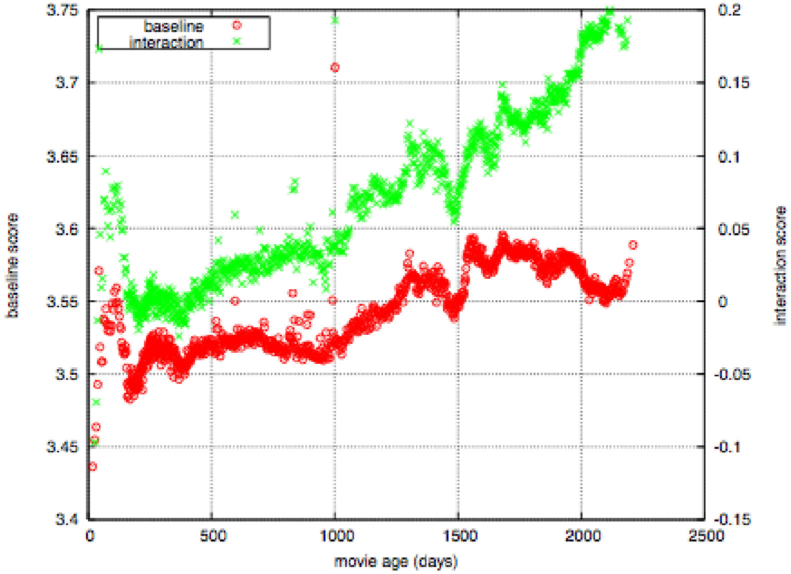
The Netflix Challenge: 2006-09

Temporal Biases Of Users

- **Sudden rise in the average movie rating** (early 2004)
  - Improvements in Netflix
  - GUI improvements
  - Meaning of rating changed
- **Movie age**
  - Users prefer new movies without any reasons
  - Older movies are just inherently better than newer ones

Y. Koren, Collaborative filtering with temporal dynamics, KDD '09





Temporal Biases & Factors

- **Original model:**

  $r_{xi} = \mu + b_x + b_i + q_i \cdot p_x$

- **Add time dependence to biases:**

  $r_{xi} = \mu + b_x(t) + b_i(t) + q_i \cdot p_x$

  - Make parameters b_x and b_i to depend on time
  - **(1)** Parameterize time-dependence by linear trends

    (2) Each bin corresponds to 10 consecutive weeks

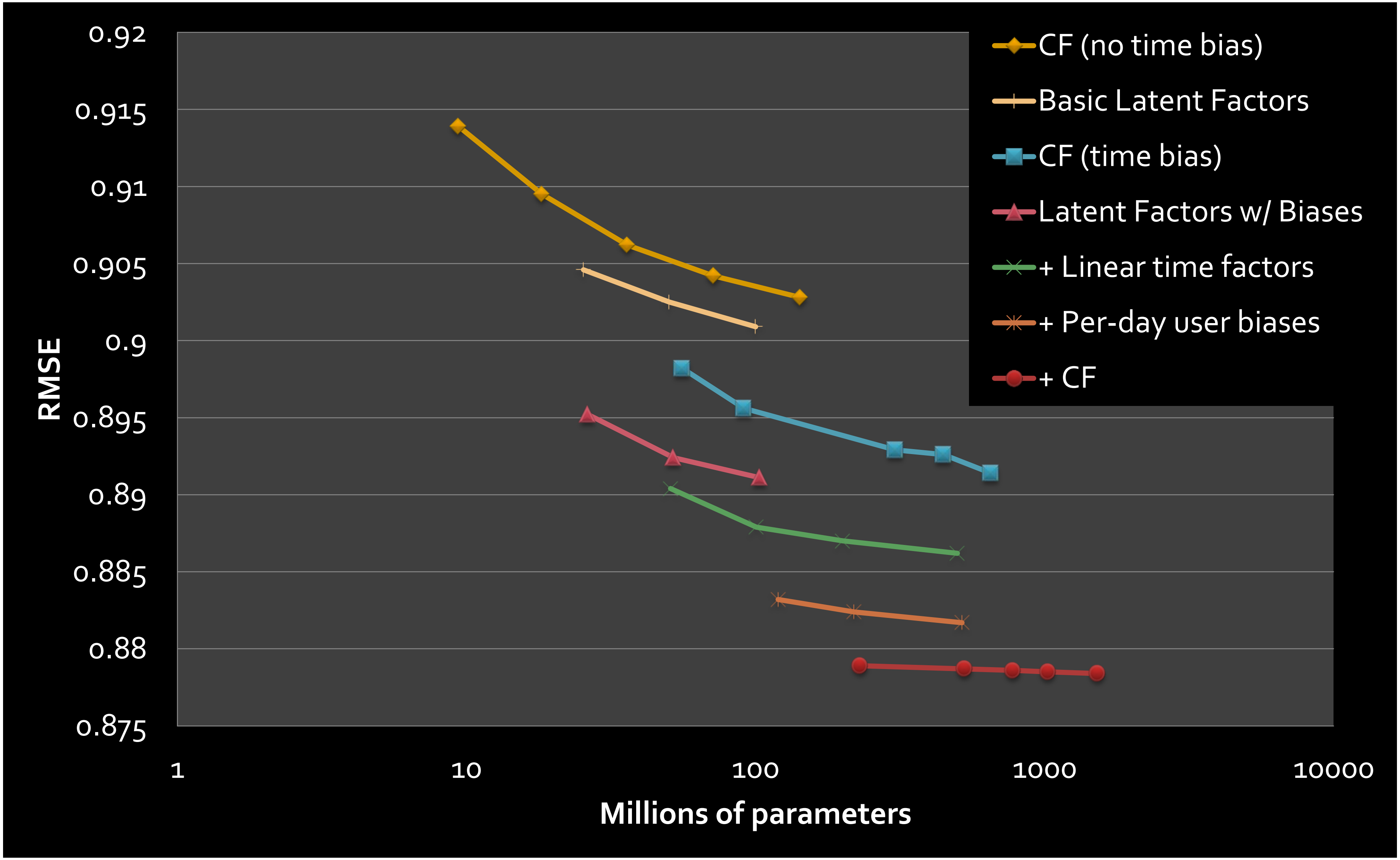
    $$b_i(t) = b_i + b_{i,\text{Bin}(t)}$$

- **Add temporal dependence to factors**

  - $p_x(t)$… user preference vector on day t

Y. Koren, Collaborative filtering with temporal dynamics, KDD ’09

Adding Temporal Effects



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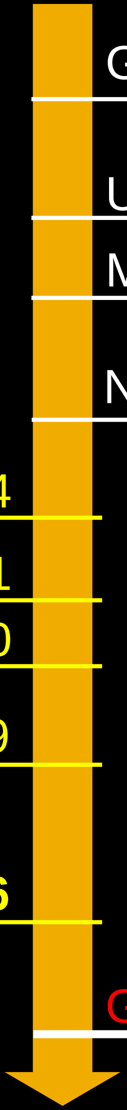

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Basic Collaborative filtering: 0.94

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Latent factors: 0.90

Latent factors+Biases: 0.89

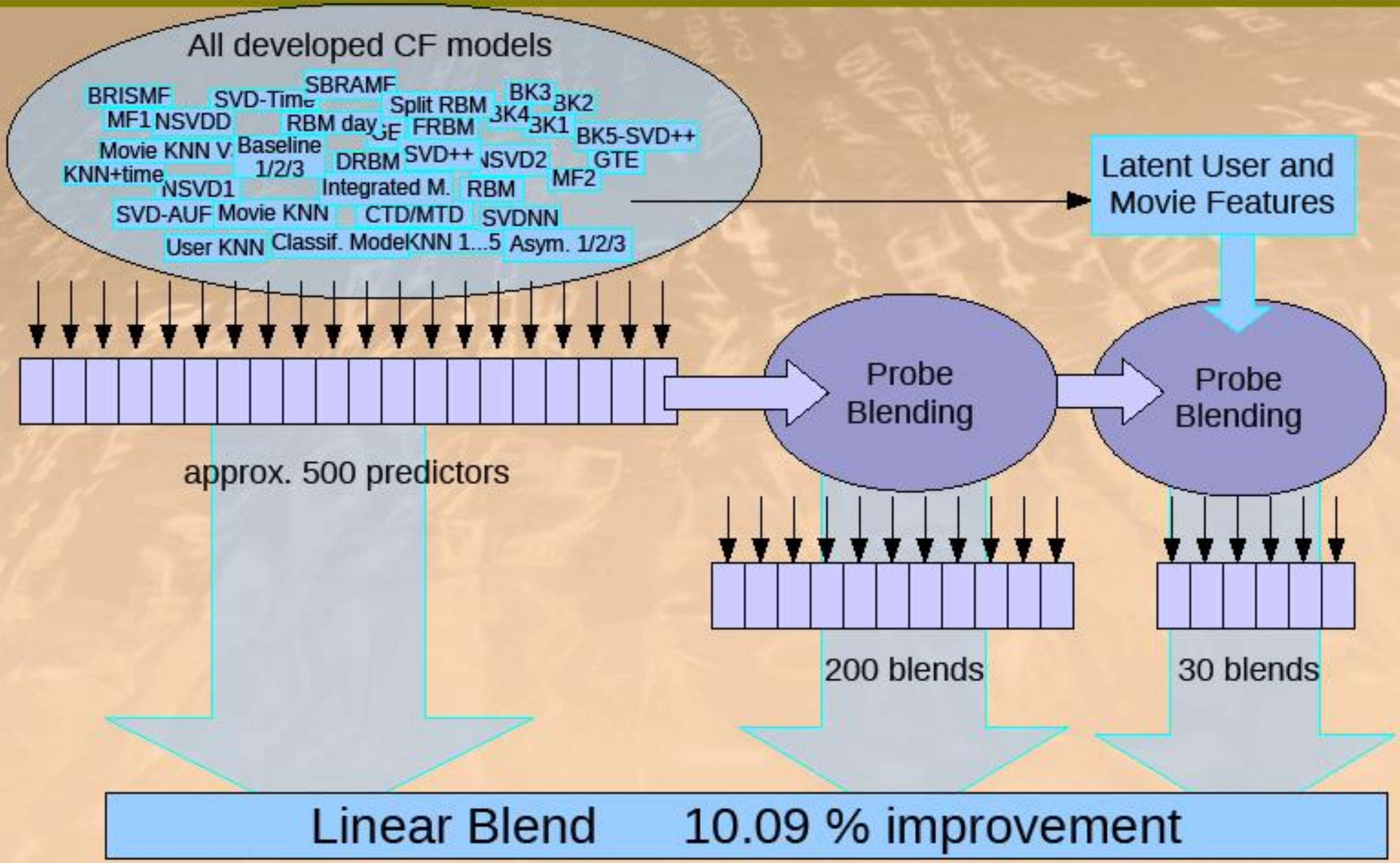
Latent factors+Biases+Time: 0.876

Grand Prize: 0.8563

Still no prize! ☹
Getting desperate.
Try a "kitchen sink" approach!

The big picture

Solution of BellKor's Pragmatic Chaos



All developed CF models

BRISMF SVD-Time SBRAMF Split RBM BK3 3K2 MF1 NSVDD RBM day GE FRBM 3K4 3K1 BK5-SVD++ Movie KNN V. Baseline 1/2/3 DRBM SVD++ NSVD2 GTE MF2 KNN+time NSVD1 Integrated M. RBM SVD-AUF Movie KNN CTD/MTD SVDNN User KNN Classif. Mode KNN 1...5 Asym. 1/2/3

Latent User and Movie Features

approx. 500 predictors

Probe Blending

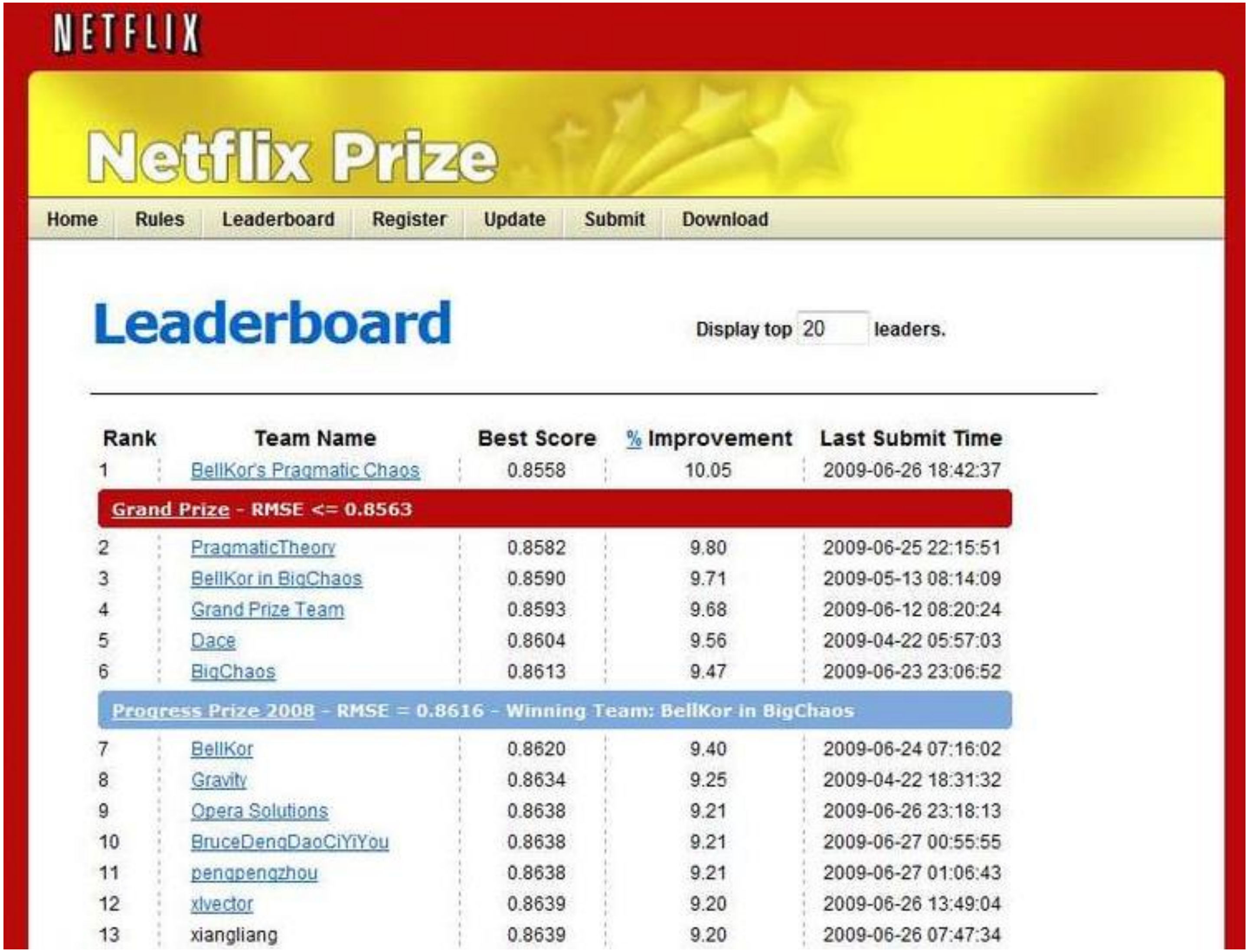
Probe Blending

200 blends

30 blends

Linear Blend 10.09 % improvement

Standing on June 26th 2009



Leaderboard

Display top 20 leaders.

Rank	Team Name	Best Score	% Improvement	Last Submit Time
1	BellKor's Pragmatic Chaos	0.8558	10.05	2009-06-26 18:42:37
Grand Prize - RMSE <= 0.8563				
2	PragmaticTheory	0.8582	9.80	2009-06-25 22:15:51
3	BellKor in BigChaos	0.8590	9.71	2009-05-13 08:14:09
4	Grand Prize Team	0.8593	9.68	2009-06-12 08:20:24
5	Dace	0.8604	9.56	2009-04-22 05:57:03
6	BigChaos	0.8613	9.47	2009-06-23 23:06:52
Progress Prize 2008 - RMSE = 0.8616 - Winning Team: BellKor in BigChaos				
7	BellKor	0.8620	9.40	2009-06-24 07:16:02
8	Gravity	0.8634	9.25	2009-04-22 18:31:32
9	Opera Solutions	0.8638	9.21	2009-06-26 23:18:13
10	BruceDengDaoCiYiYou	0.8638	9.21	2009-06-27 00:55:55
11	pengpengzhou	0.8638	9.21	2009-06-27 01:06:43
12	xlvector	0.8639	9.20	2009-06-26 13:49:04
13	xiangliang	0.8639	9.20	2009-06-26 07:47:34

June 26th submission triggers 30-day "last call"

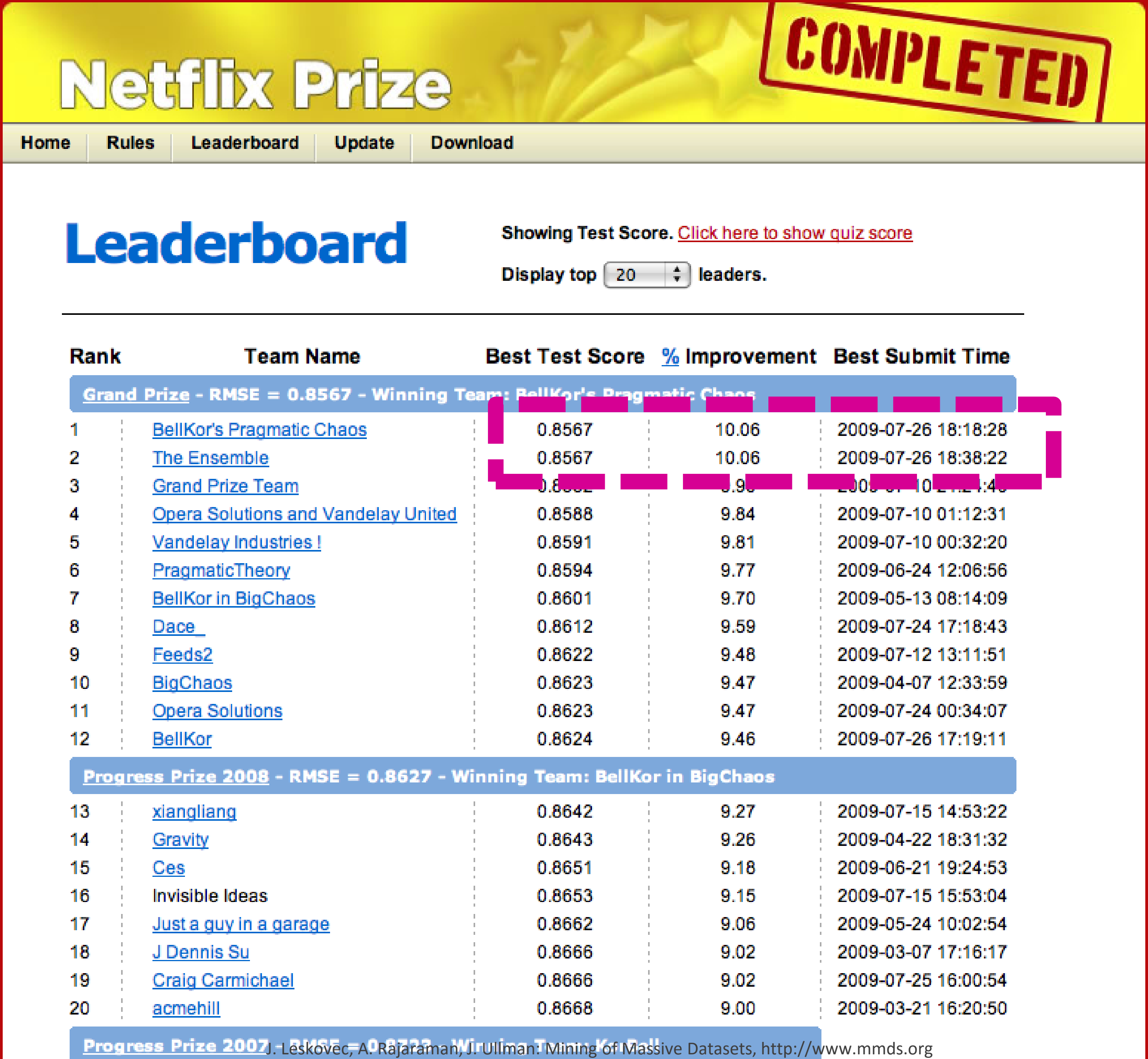
The Last 30 Days

- **Ensemble team formed**
  - Group of other teams on leaderboard forms a new team
  - Relies on combining their models
  - Quickly also get a qualifying score over 10%
- **BellKor**
  - Continue to get small improvements in their scores
  - Realize that they are in direct competition with Ensemble
- **Strategy**
  - Both teams carefully monitoring the leaderboard
  - Only sure way to check for improvement is to submit a set of predictions
    - This alerts the other team of your latest score

24 Hours from the Deadline

- **Submissions limited to 1 a day**
  - Only 1 final submission could be made in the last 24h
- **24 hours before deadline…**
  - **BellKor** team member in Austria notices (by chance) that **Ensemble** posts a score that is slightly better than BellKor's
- **Frantic last 24 hours for both teams**
  - Much computer time on final optimization
  - Carefully calibrated to end about an hour before deadline
- **Final submissions**
  - **BellKor** submits a little early (on purpose), 40 mins before deadline
  - **Ensemble** submits their final entry 20 mins later
  - **….and everyone waits….**

NETFLIX



Netflix Prize

Leaderboard

Showing Test Score. Click here to show quiz score

Display top 20 leaders.

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	BellKor's Pragmatic Chaos	0.8567	10.06	2009-07-26 18:18:28
2	The Ensemble	0.8567	10.06	2009-07-26 18:38:22
3	Grand Prize Team	[illegible]	[illegible]	[illegible]
4	Opera Solutions and Vandelay United	0.8588	9.84	2009-07-10 01:12:31
5	Vandelay Industries !	0.8591	9.81	2009-07-10 00:32:20
6	PragmaticTheory	0.8594	9.77	2009-06-24 12:06:56
7	BellKor in BigChaos	0.8601	9.70	2009-05-13 08:14:09
8	Dace_	0.8612	9.59	2009-07-24 17:18:43
9	Feeds2	0.8622	9.48	2009-07-12 13:11:51
10	BigChaos	0.8623	9.47	2009-04-07 12:33:59
11	Opera Solutions	0.8623	9.47	2009-07-24 00:34:07
12	BellKor	0.8624	9.46	2009-07-26 17:19:11
Progress Prize 2008 - RMSE = 0.8627 - Winning Team: BellKor in BigChaos				
13	xiangliang	0.8642	9.27	2009-07-15 14:53:22
14	Gravity	0.8643	9.26	2009-04-22 18:31:32
15	Ces	0.8651	9.18	2009-06-21 19:24:53
16	Invisible Ideas	0.8653	9.15	2009-07-15 15:53:04
17	Just a guy in a garage	0.8662	9.06	2009-05-24 10:02:54
18	J Dennis Su	0.8666	9.02	2009-03-07 17:16:17
19	Craig Carmichael	0.8666	9.02	2009-07-25 16:00:54
20	acmehill	0.8668	9.00	2009-03-21 16:20:50

Progress Prize 2007 [illegible]

Million $ Awarded Sept 21st 2009

2009

DATE 09 21 09

PAY TO THE ORDER OF Bellkor's Pragmatic Chaos

$ 1,000,000 00

AMOUNT ONE MILLION 00/100

FOR The Netflix Prize

Reed Hastings

Acknowledgments

- Some slides and plots borrowed from Yehuda Koren, Robert Bell and Padhraic Smyth
- **Further reading:**
  - Y. Koren, Collaborative filtering with temporal dynamics, KDD ’09
  - http://www2.research.att.com/~volinsky/netflix/bpc.html
  - http://www.the-ensemble.com/