CS-473: Algorithms I Spring 2018

Assignment 2: Asymptotic Notation

Instructor: Mehmet Koyutürk

Due Date and Instructions

Please return hard-copies at the beginning of the class meeting (10:40) on **Tuesday, February 20, 2018**.

- Hand-writing is accepted but the course personnel reserves the right to reject grading an assignment if the hand-writing is not legible.
- Assignments written in LaTeX will receive 5 bonus points.
- You can submit the assignment as a team of 2 students, however no pair of students are allowed to work together on more than 3 assignments.
- You are also allowed to talk to other students and the course personnel about the solutions, but you must write the answers yourself. Answers copied from other students or resources will be detected, and appropriate action will be taken.

Problem 1

Considering functions $f(n) \geq 0$, $g(n) \geq 0$, and constant $c > 0$, indicate whether each of the following statements is true. Prove the statements that are true by providing a formal argument that is based on the definition of asymptotic notation. For statements that are false, provide a counter-example to prove that they are false.

(a) [10 pts] $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$.

(b1) [8 pts] $f(n) + c = O(f(n))$.

(b2) [8 pts] If $f(n) \geq 1$, then $f(n) + c = O(f(n))$.

(c1) [8 pts] If $f(n) = O(g(n))$, $\log(f(n)) \geq 0$ and $\log(g(n)) \geq 0$, then $\log(f(n)) = O(\log(g(n)))$.

(c2) [8 pts] If $f(n) = O(g(n))$, $\log(f(n)) \geq 0$ and $\log(g(n)) \geq 1$, then $\log(f(n)) = O(\log(g(n)))$.

(d1) [6 pts] $f(2n) = \Theta(f(n))$.

(d2) [6 pts] If $f(n) = O(n^c)$, then $f(2n) = O(n^c)$.

(d3) [6 pts] If $f(n) = \Theta(n^c)$, then $f(2n) = \Theta(f(n))$.

Problem 2

[40 pts] Assuming that $0 < \epsilon < 1 < a < b$ are constants, sort the following functions in asymptotically increasing order. Indicate when two or more functions are asymptotically equivalent. Briefly justify your answers (no formal proof is needed).

n^ϵ	ϵ^n	a^n	b^n
$a^{\log_a n}$	$\log(n^a)$	$\log(n^b)$	n/a
ϵn	$(n+a)^b$	n^{a+b}	$(n+b)^a$
$n^{-\epsilon}$	n^{-a}	n^a	$\log(n^\epsilon)$
$\log_{1/\epsilon} n$	$(\log n)^a$	$\log(bn)$	$a^{\epsilon n}$