

GE 461 Introduction to Data Science

Spring 2025

Deep Learning

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Slide Credits: F. Li, A. Karpathy, J. Johnson, G. Cinbis

So, What is DEEP Machine Learning

A few different ideas:

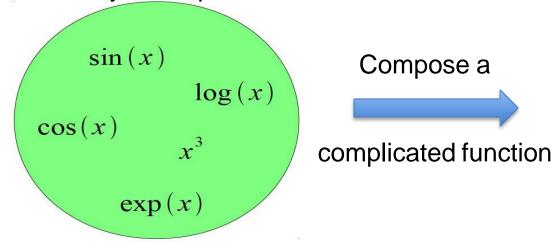
- (Hierarchical) Compositionality
 - Cascade of non-linear transformations
 - Multiple layers of representations
- End-to-End Learning
 - Learning (goal-driven) representations
 - Learning feature extraction
- Distributed Representations
 - No single neuron "encodes" everything
 - Groups of neurons work together

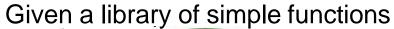
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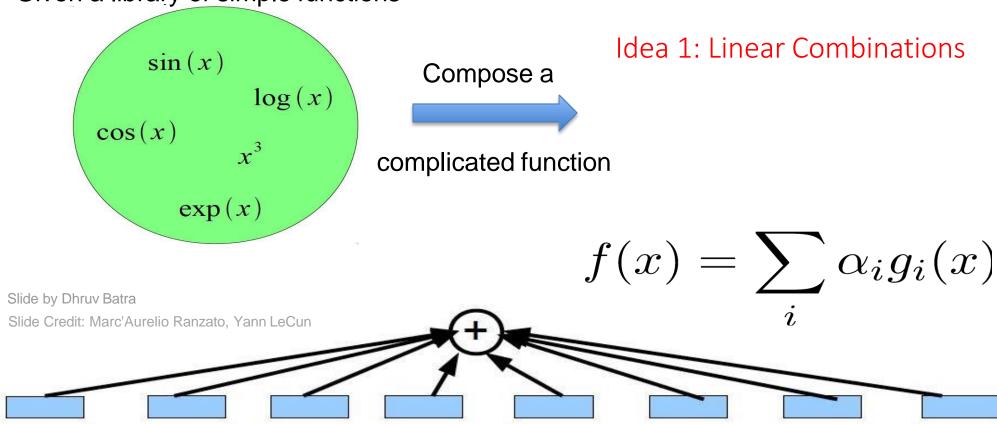
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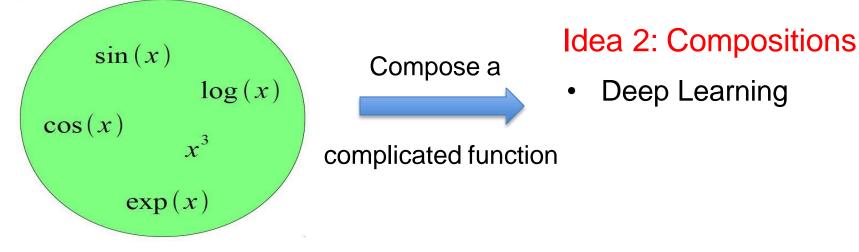
Given a library of simple functions







Given a library of simple functions



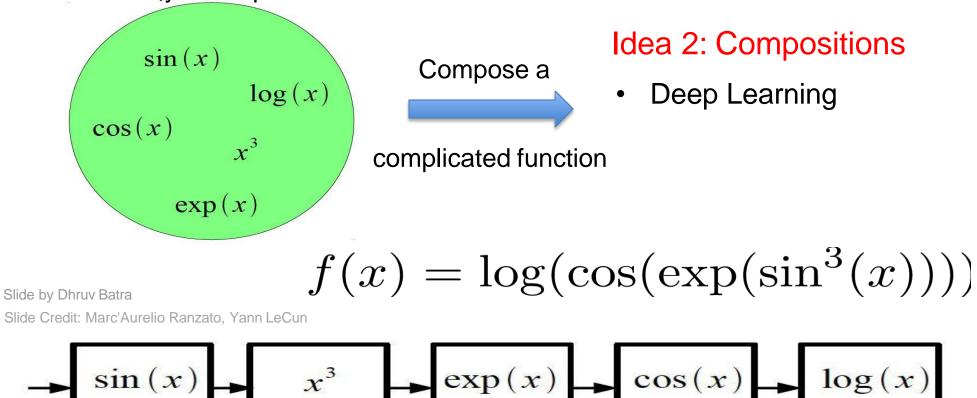
Slide by Dhruv Batra

Slide Credit: Marc'Aurelio Ranzato, Yann LeCun

$$f(x) = g_1(g_2(\dots(g_n(x)\dots))$$

Given a library of simple functions

 $\sin(x)$

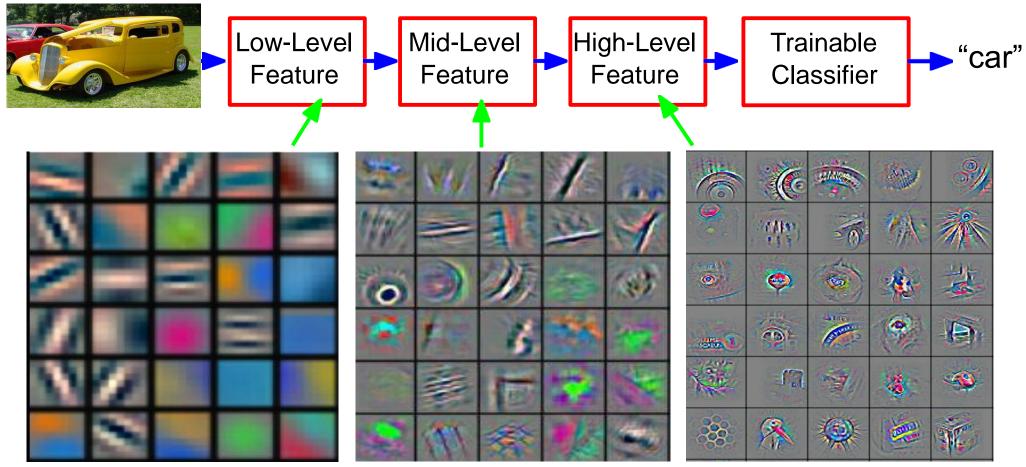


 $\exp(x)$

 $\cos(x)$

 $\log(x)$

Deep Learning = Hierarchical Compositionality



Slide by Dhruv Batra

Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

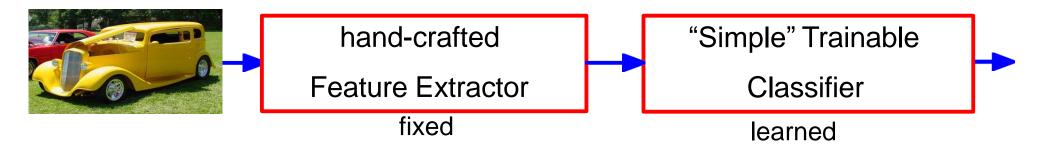
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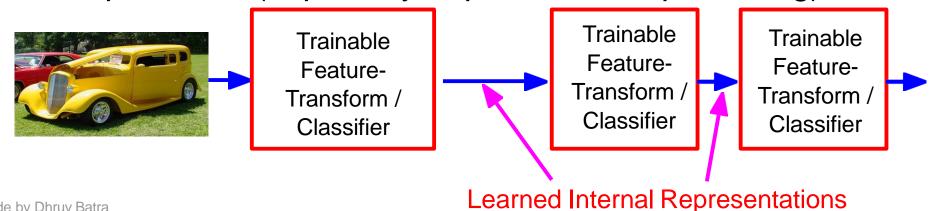
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"Shallow" vs Deep Learning

"Shallow" models



Deep models (especially supervised deep learning)



Slide by Dhruv Batra Slide Credit: Marc'Aurelio Ranzato, Yann LeCun

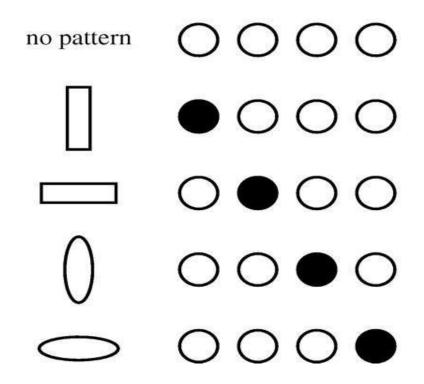
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Distributed Representations Toy Example

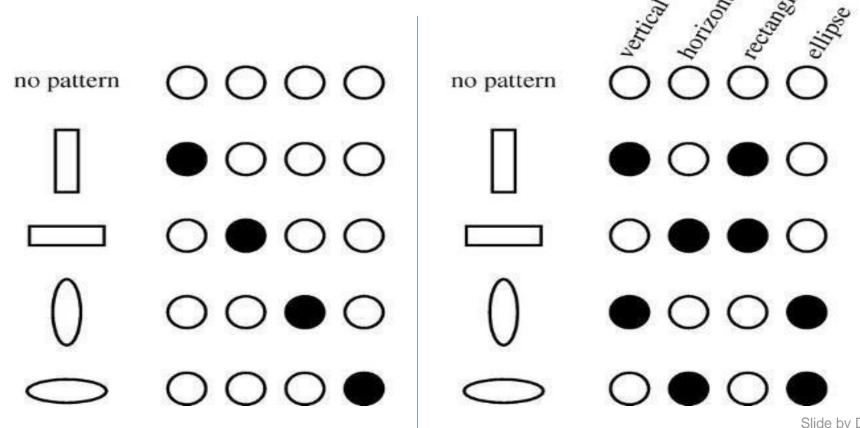
Local vs Distributed



Slide by Dhruv Batra Slide Credit: Moontae Lee

Distributed Representations Toy Example

Can we interpret each dimension?



Slide by Dhruv Batra Slide Credit: Moontae Lee

Power of distributed representations!

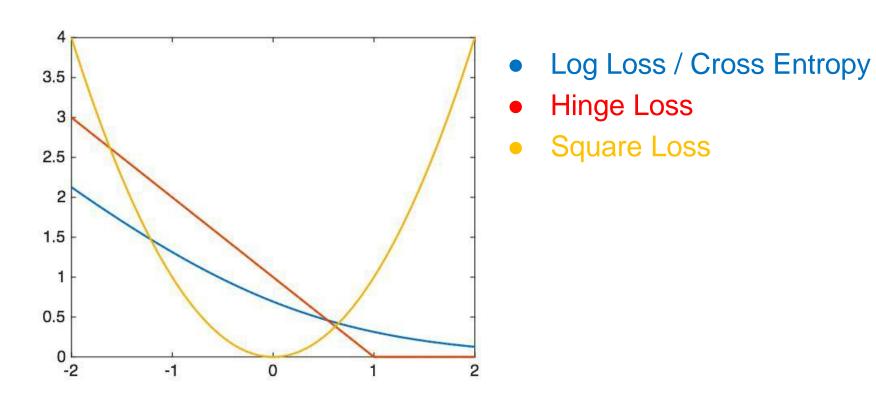
$$f(x, W) = Wx$$

- Loss function
- Optimization
- Convolutional Nets
- Recurrent Nets

Loss Functions

Loss functions

• There are many different loss functions



Classification Losses

Hinge Loss/Multi class SVM Loss

$$SVMLoss = \sum_{j \neq y_i} max(0, s_j - s_{y_i} + 1)$$

- s_j Computed score of the training example for jth class.
- y(i) Ground truth label for ith training example.

Classification Losses

Cross Entropy Loss/Negative Log Likelihood

$$CrossEntropyLoss = -\log(\frac{e^{sy_i}}{\sum_j e^{s_j}})$$

- s_j Computed score of the training example for jth class.
- y(i) Ground truth label for ith training example.

Regression Losses

Mean Square Error/Quadratic Loss/L2 Loss

$$MSE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}$$

- n Number of training examples.
- i ith training example in a data set.
- y(i) Ground truth label for ith training example.
- y_hat(i) Prediction for ith training example.

Regression Losses

Mean Absolute Error/L1 Loss

$$MAE = \frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{n}$$

- n Number of training examples.
- i ith training example in a data set.
- y(i) Ground truth label for ith training example.
- y_hat(i) Prediction for ith training example.

Regression Losses

Mean Bias Error

$$MBE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)}{n}$$

- n Number of training examples.
- i ith training example in a data set.
- y(i) Ground truth label for ith training example.
- y_hat(i) Prediction for ith training example.

Weight Regularization

$$L = \frac{1}{N} \sum_{i=1}^{N} \operatorname{Loss}_{i} + \lambda R(W)$$

\lambda = regularization strength (hyperparameter)

Some reg. types:

L2 regularization

L1 regularization

Elastic net (L1 + L2)

$$R(W) = \sum_{k} \sum_{l} W_{k,l}^2$$

$$R(W) = \sum_k \sum_l |W_{k,l}|$$

$$R(W) = \sum_k \sum_l eta W_{k,l}^2 + |W_{k,l}|$$

. . .

L2 regularization: motivation

$$egin{aligned} x &= [1,1,1,1] \ & w_1 &= [1,0,0,0] \ & w_2 &= [0.25,0.25,0.25,0.25] \end{aligned}$$

$$w_1^Tx=w_2^Tx=1$$

L2 regularization: motivation

$$x = [1, 1, 1, 1]$$

$$w_1 = [1, 0, 0, 0]$$

$$w_2 = [0.25, 0.25, 0.25, 0.25]$$

Which one does L2 regularization choose?

$$w_1^Tx=w_2^Tx=1$$

L2 regularization: motivation

$$x = [1, 1, 1, 1]$$

$$w_1 = [1, 0, 0, 0]$$

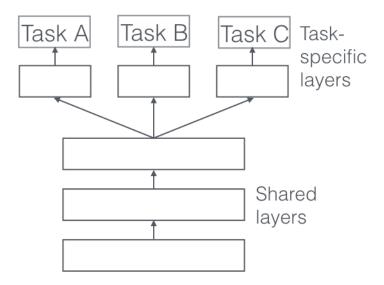
$$w_2 = [0.25, 0.25, 0.25, 0.25]$$

Why does it make sense?

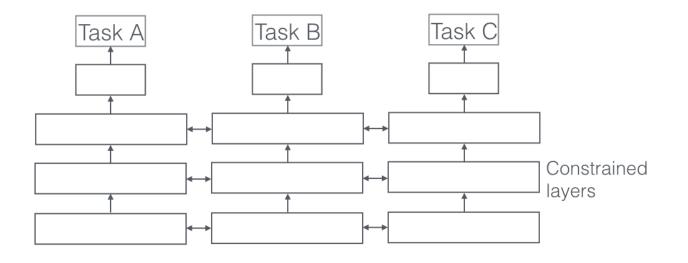
$$w_1^Tx=w_2^Tx=1$$

Multi-task Learning

Jointly minimize the losses of different tasks



Hard parameter sharing for multi-task learning in deep neural networks

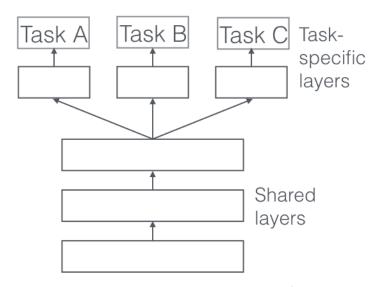


Soft parameter sharing for multi-task learning in deep neural networks

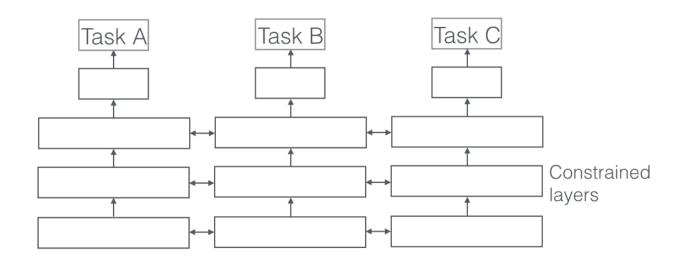
Multi-task Learning

Jointly minimize the losses of different tasks (combine loss terms)

$$L = l_a + \alpha l_b + \beta l_c + \cdots$$



Hard parameter sharing for multi-task learning in deep neural networks



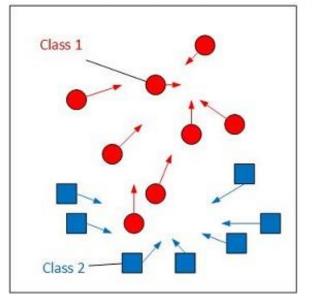
Soft parameter sharing for multi-task learning in deep neural networks

Metric/Contrastive Learning

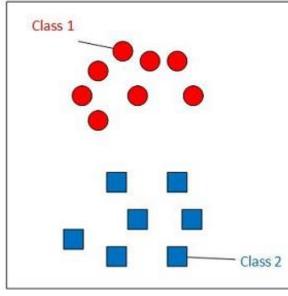
Learn distinctiveness

- 1.A distance-based loss function (as opposed to prediction error-based loss functions like Logistic loss or Hinge loss used in Classification).
- 2.Like any distance-based loss, it tries to ensure that semantically similar examples are embedded close together.
- 3.Defined based on pairs (+/-class pairs) or groups of samples.

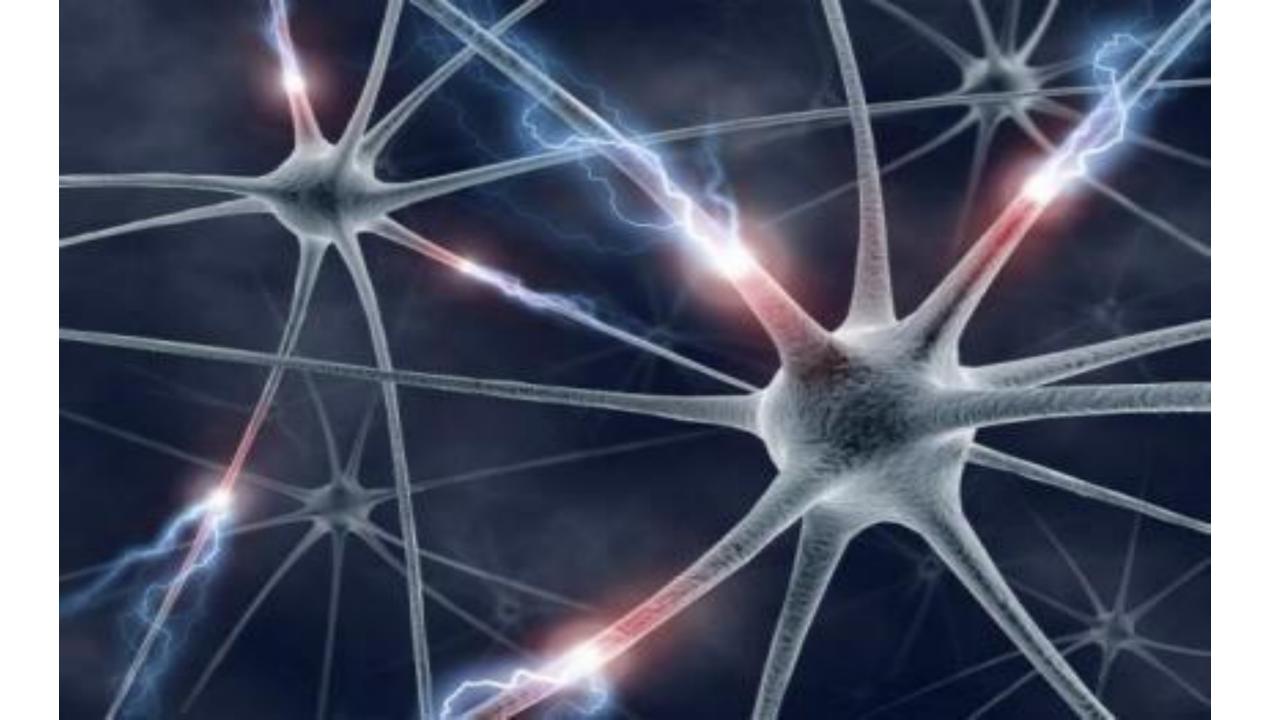
Before Metric Learning



After Metric Learning



$$L_i = \sum_{i \neq j} \| w^T x_{i,c1} - w^T x_{j,c1} \| - \sum_{k} \| w^T x_{i,c1} - w^T x_{k,c2} \|$$



Neural Networks

Linear score function:

2-layer Neural Network:

$$f = Wx$$

$$f = W_2 \max(0, W_1 x)$$

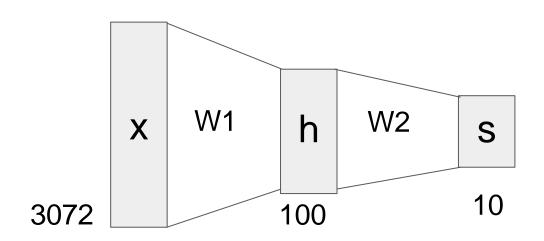
Neural Networks

Linear score function:

$$f = Wx$$

2-layer Neural Network:

$$f = W_2 \max(0, W_1 x)$$



Neural Networks

Linear score function:

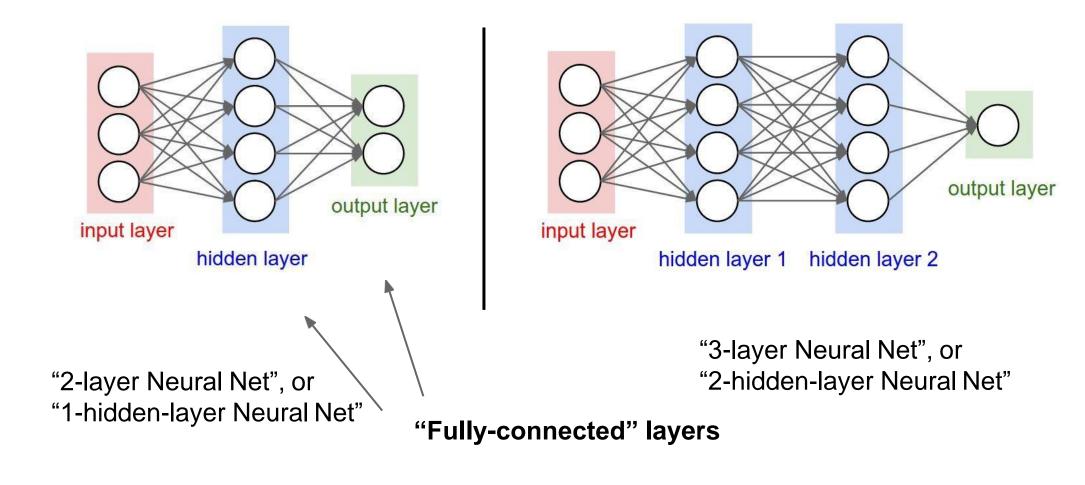
$$f = Wx$$

2-layer Neural Network or 3-layer Neural Network

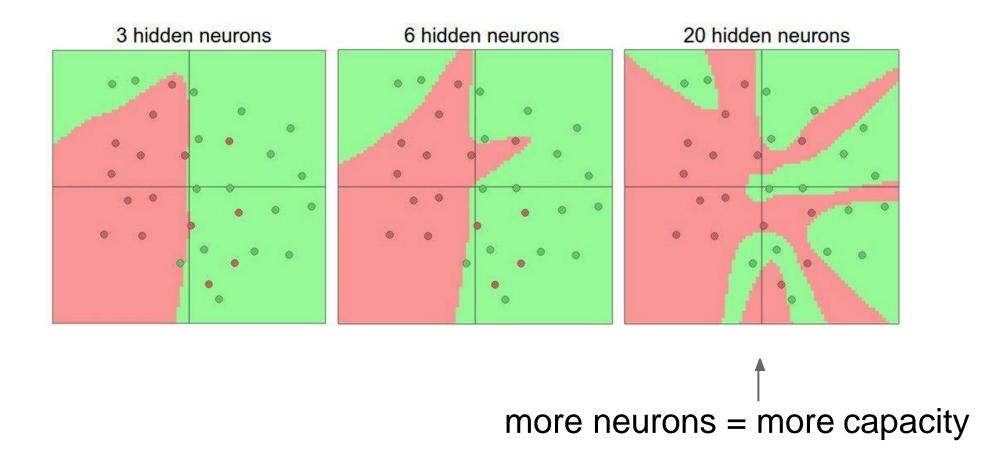
$$f = W_2 \max(0, W_1 x)$$

$$f = W_3 \max(0, W_2 \max(0, W_1 x))$$

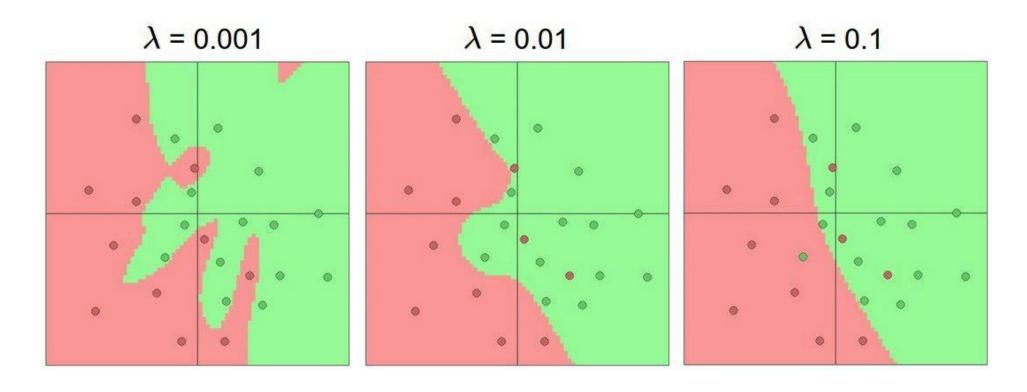
Neural Networks: Architectures



Setting the number of layers and their sizes

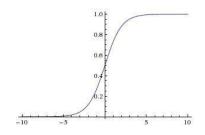


Do not use size of neural network as a regularizer. Use stronger regularization instead:

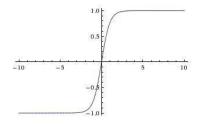


Sigmoid

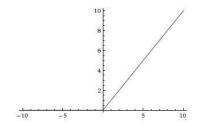
$$\sigma(x) = 1/(1 + e^{-x})$$



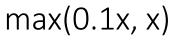
tanh tanh(x)

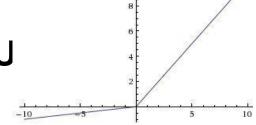


ReLU max(0,x)



Leaky ReLU





Maxout

$$\max(w_1^Tx+b_1,w_2^Tx+b_2)$$

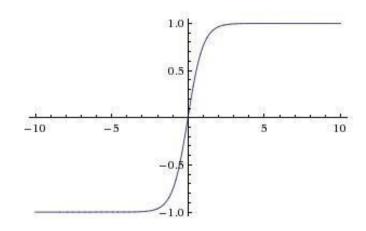
Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

3 problems:

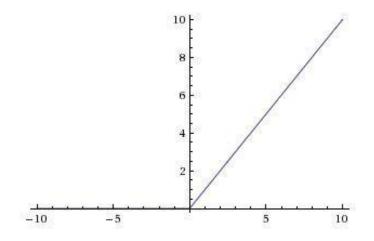
- 1. Saturated neurons "kill" the gradients
- 2. Sigmoid outputs are not zero-centered
- 3. exp() is a bit computationally expensive



$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated:(

[LeCun et al., 1991]



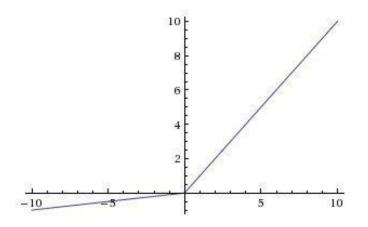
ReLU (Rectified Linear Unit)

- Computes f(x) = max(0,x)
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Not zero-centered output
- An annoyance:

hint: what is the gradient when x < 0?

[Krizhevsky et al., 2012]

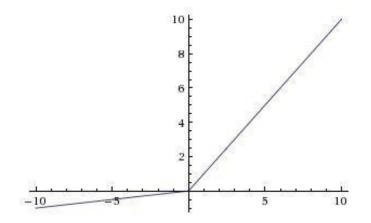
[Mass et al., 2013] [He et al., 2015]



- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- will not "die".

Leaky ReLU

$$f(x) = \max(0.01x, x)$$



Leaky ReLU

$$f(x) = \max(0.01x, x)$$

- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- will not "die".

Parametric Rectifier (PReLU)

$$f(x) = \max(\alpha x, x)$$

backprop into \alpha (parameter)

- Does not have the basic form of dot product -> nonlinearity
- Generalizes ReLU and Leaky ReLU
- Linear Regime! Does not saturate! Does not die!

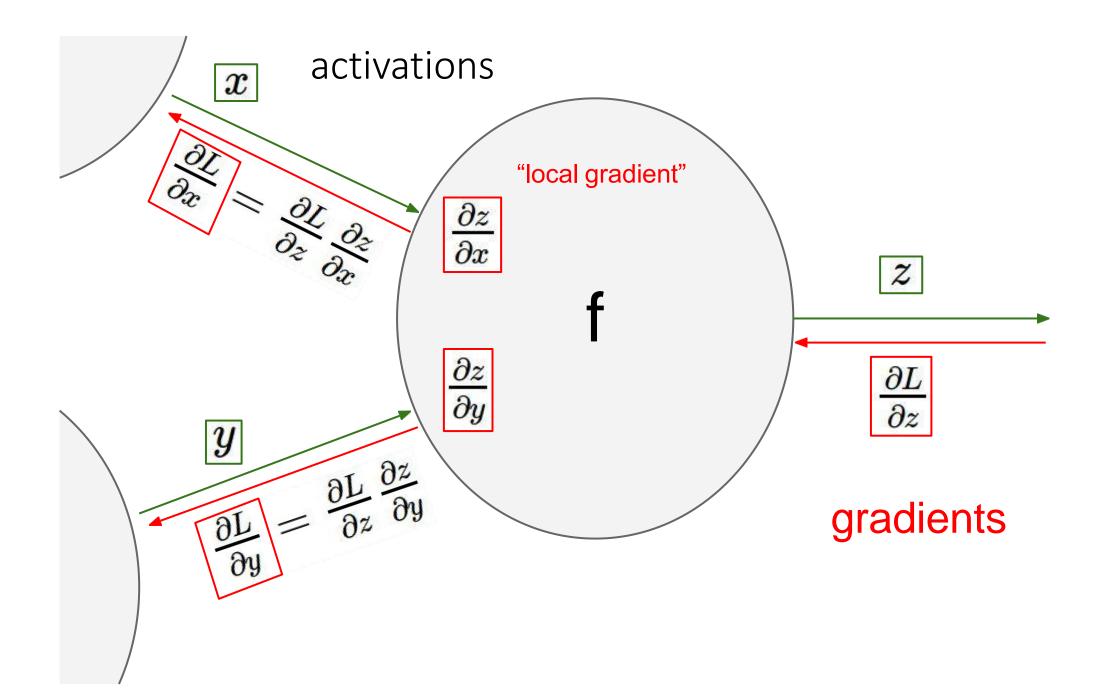
$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

Problem: doubles the number of parameters/neuron:(

In practice

- Use ReLU. Be careful with your learning rates
- Try out Leaky ReLU / Maxout
- Try out tanh but don't expect much
- Don't use sigmoid

Parameter Updates



Training a neural network, main loop:

```
while True:
    data_batch = dataset.sample_data_batch()
    loss = network.forward(data_batch)
    dx = network.backward()
    x += - learning_rate * dx
```

simple gradient descent update

Optimize the parameters using one of the SGD variants

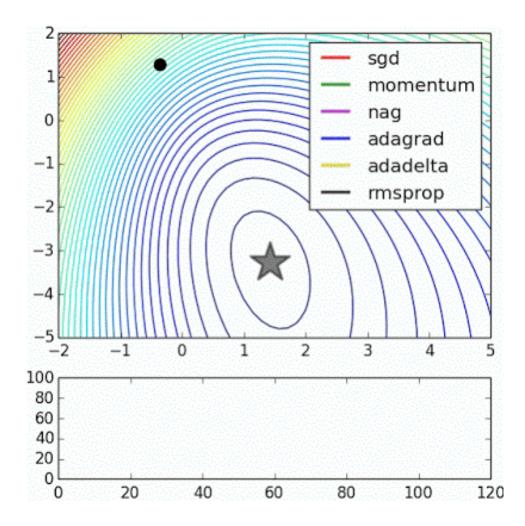
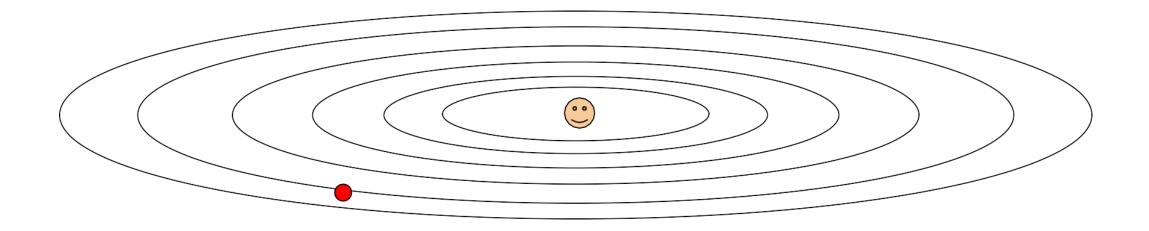


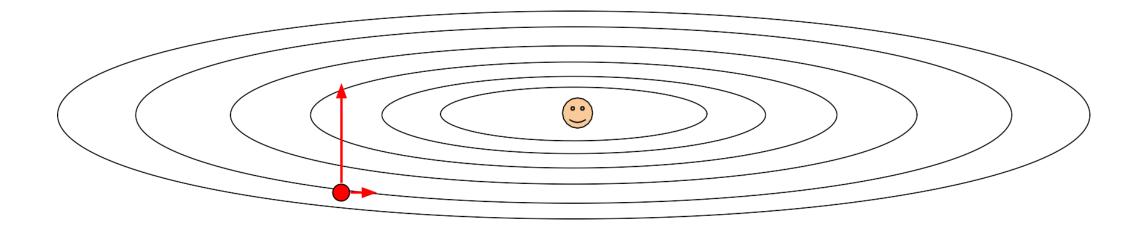
Image credits: Alec Radford

Suppose loss function is steep vertically but shallow horizontally:



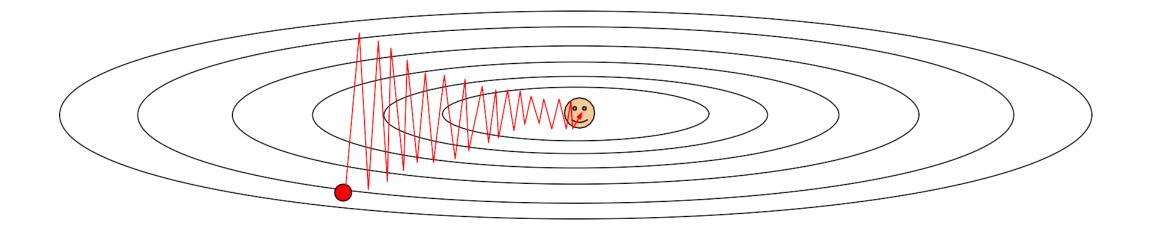
Q: What is the trajectory along which we converge towards the minimum with SGD?

Suppose loss function is steep vertically but shallow horizontally:



Q: What is the trajectory along which we converge towards the minimum with SGD?

Suppose loss function is steep vertically but shallow horizontally:



Q: What is the trajectory along which we converge towards the minimum with SGD? very slow progress along flat direction, jitter along steep one

Momentum Update

```
# Gradient descent update
x += - learning_rate * dx

# Momentum update
v = mu * v - learning_rate * dx # integrate velocity
x += v # integrate position
```

- Physical interpretation as ball rolling down the loss function + friction (mu coefficient).
- μ μ

Momentum Update

```
# Gradient descent update
x += - learning_rate * dx

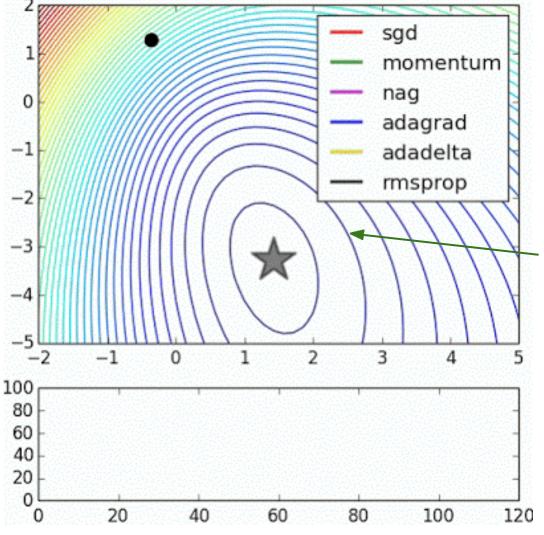
# Momentum update
v = mu * v - learning_rate * dx # integrate velocity
x += v # integrate position
```

- Allows a velocity to "build up" along shallow (yet consistent) directions
- Velocity becomes damped in steep (inconsistent) direction due to quickly changing sign

SGD

VS

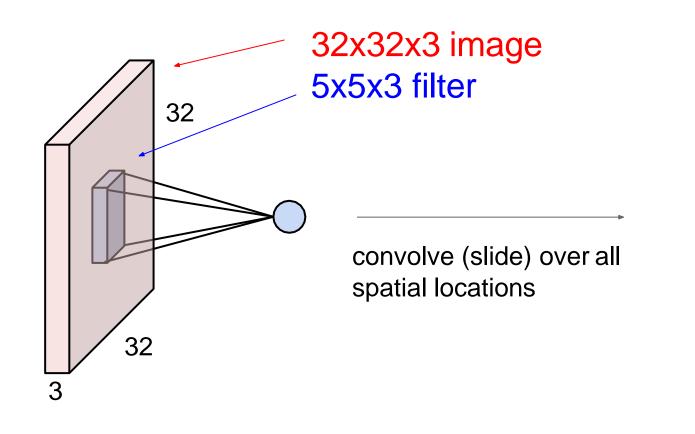
Momentum



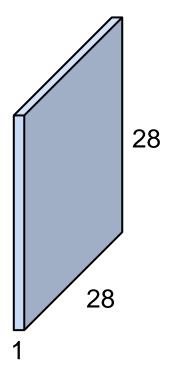
notice momentum overshooting the target, but overall getting to the minimum much faster.

Convolutional Neural Networks

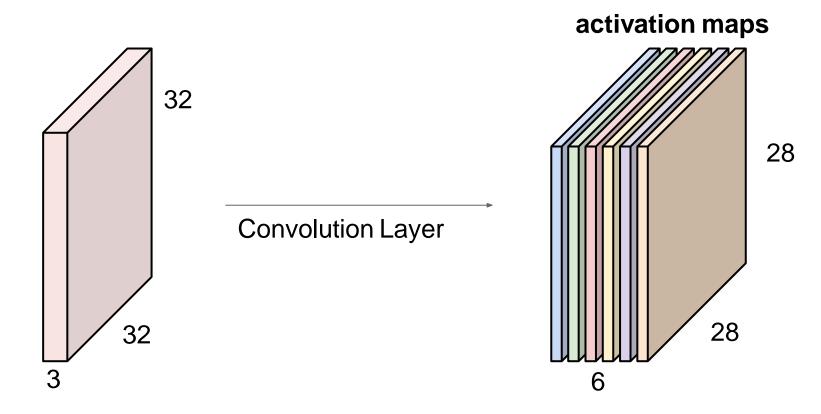
Convolution Layer



activation map

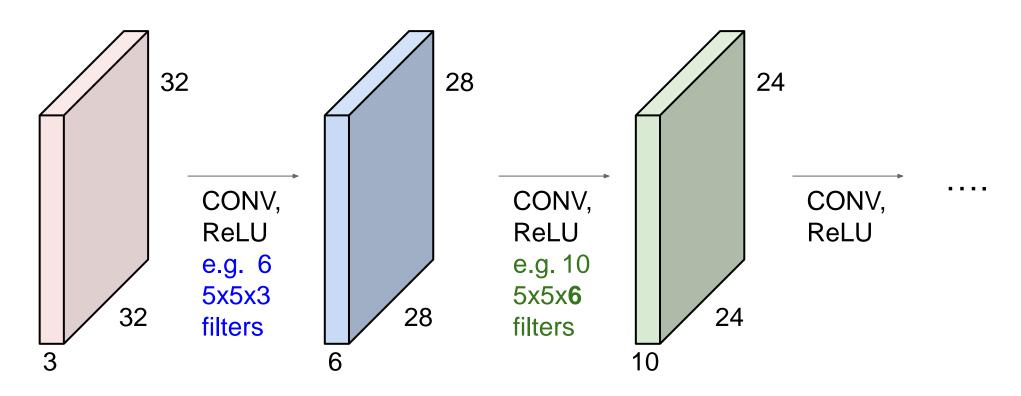


For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:



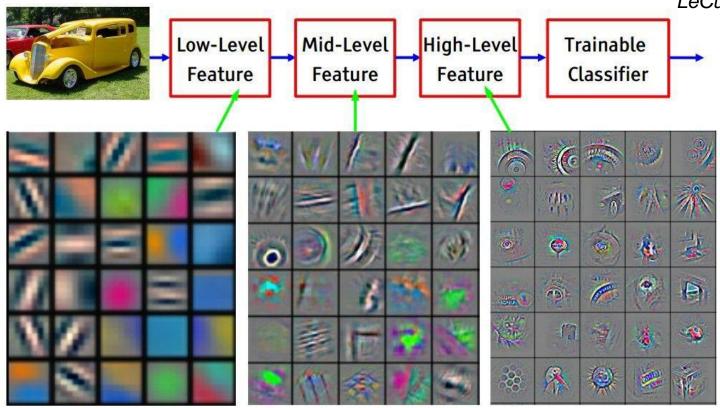
We stack these up to get a "new image" of size 28x28x6!

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



Preview:

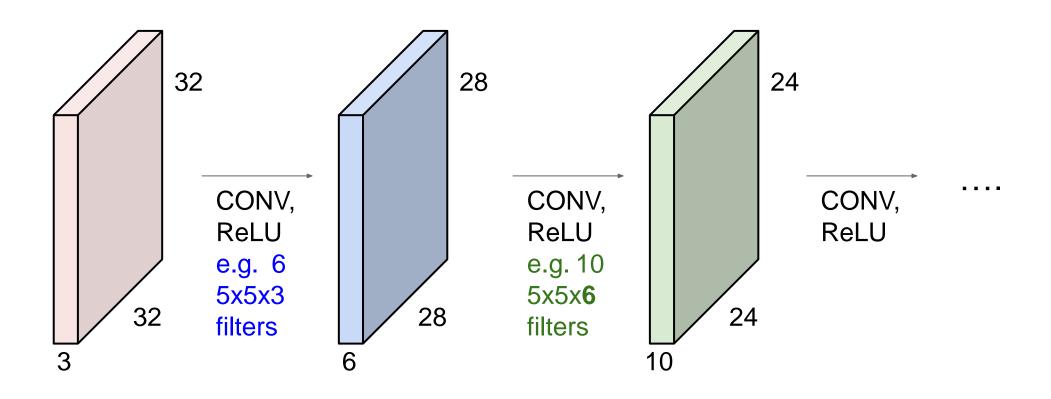
[From recent Yann LeCun slides]



Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

Remember back to...

E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially! $(32 \rightarrow 28 \rightarrow 24 \dots)$. Shrinking too fast is not good, doesn't work well.

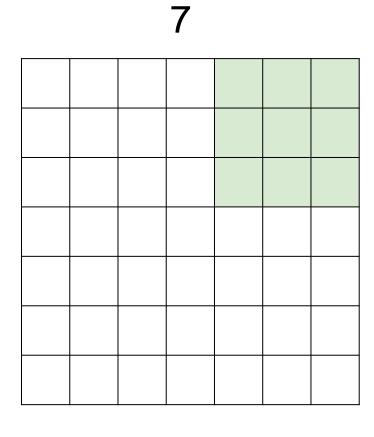


7

7

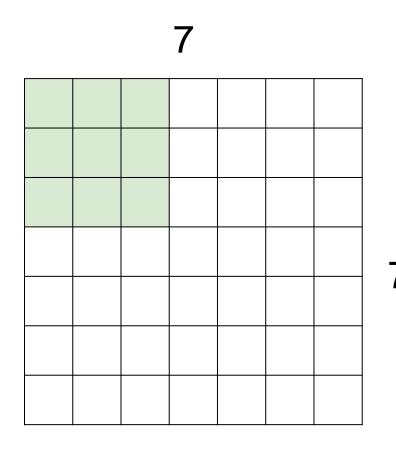
7

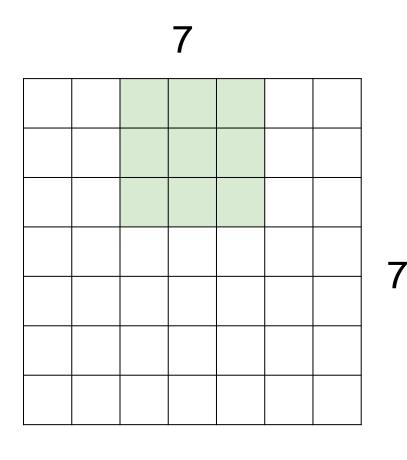
7

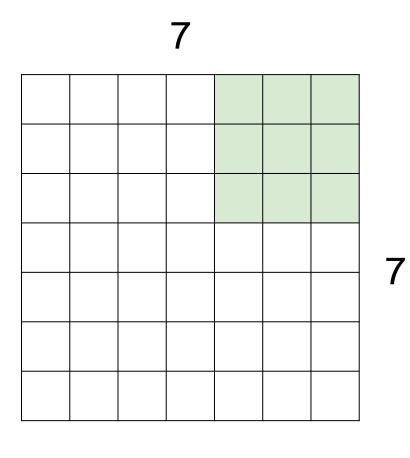


7x7 input (spatially) assume 3x3 filter applied with stride 1

→ 5x5 output







7x7 input (spatially) assume 3x3 filter applied with stride 2 → 3x3 output!

| ı | N I | |
|---|------------|--|
| ı | \ I | |
| ı | V | |
| | | |

| | F | | |
|---|---|--|--|
| | | | |
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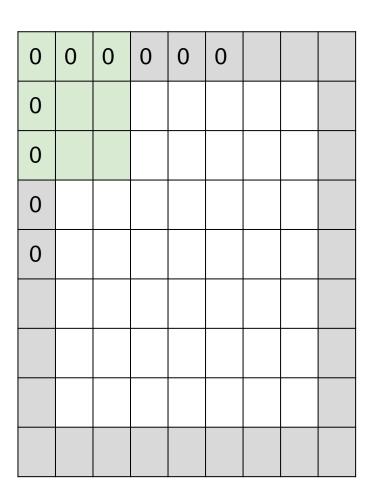
Output size:

(N - F) / stride + 1

e.g. N = 7, F = 3: stride $1 \Rightarrow (7 - 3)/1 + 1 = 5$ stride $2 \Rightarrow (7 - 3)/2 + 1 = 3$

. . .

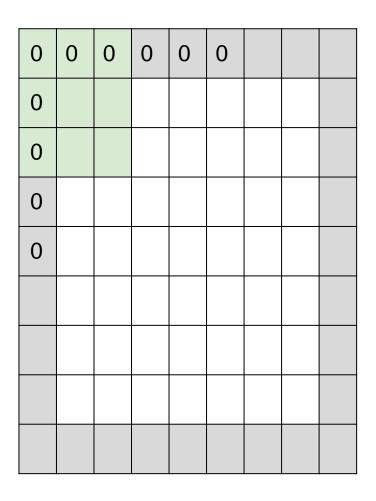
In practice: Common to zero pad the border



e.g. input 7x7
3x3 filter, applied with stride 1
pad with 1 pixel border => what is the output?

```
(recall:)
(N - F) / stride + 1
```

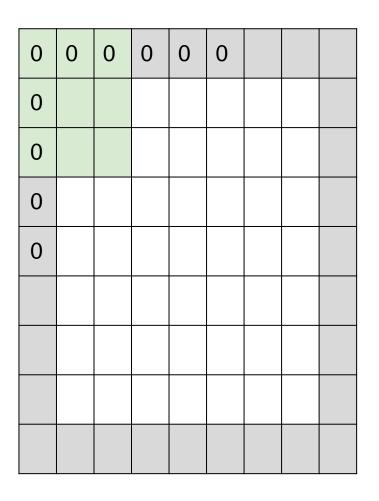
In practice: Common to zero pad the border



e.g. input 7x73x3 filter, applied with stride 1pad with 1 pixel border => what is the output?

7x7 output!

In practice: Common to zero pad the border

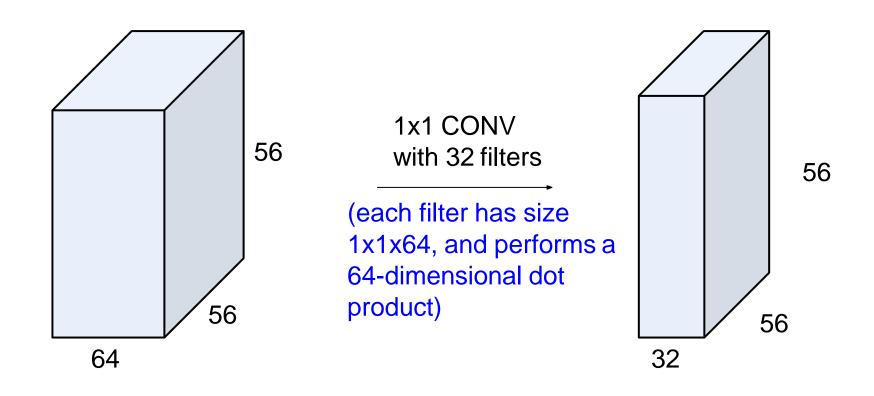


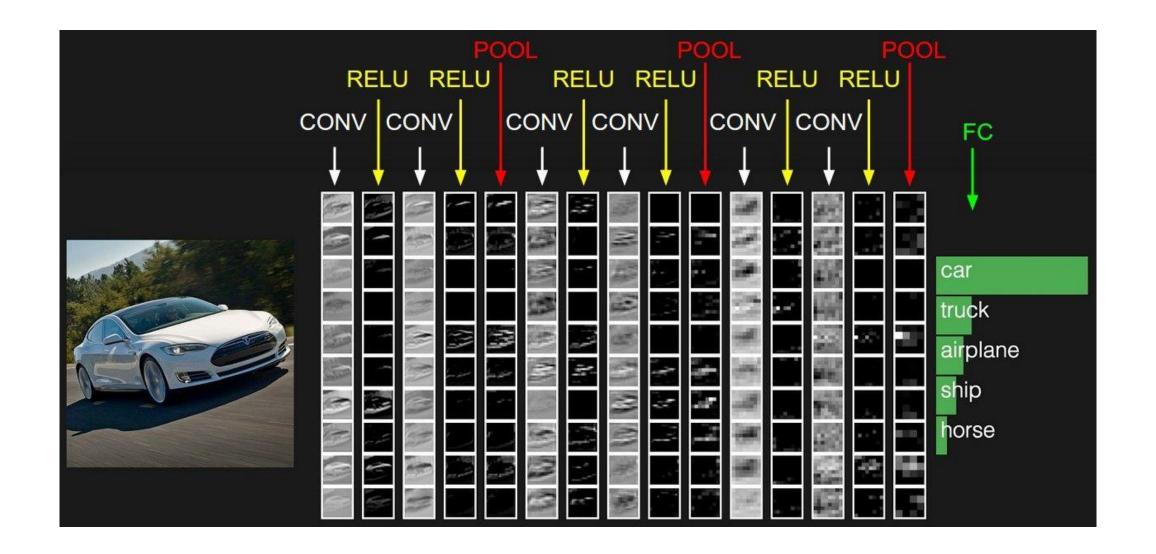
e.g. input 7x7
3x3 filter, applied with stride 1
pad with 1 pixel border => what is the output?

In general, common to see CONV layers with stride 1, filters of size FxF, and zero-padding with (F-1)/2. (will preserve size spatially)

```
e.g. F = 3 => zero pad with 1
F = 5 => zero pad with 2
F = 7 => zero pad with 3
```

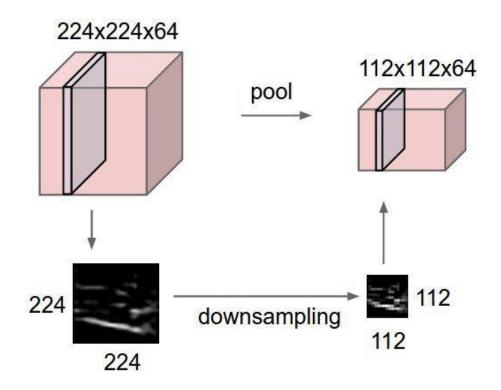
1x1 convolution layers





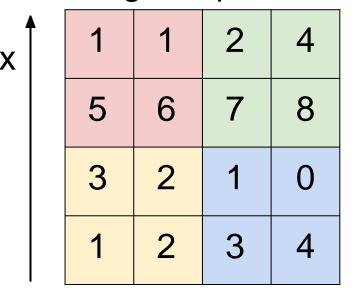
Pooling layer

- makes the representations smaller and more manageable
- operates over each activation map independently:



MAX Pooling

Single depth slice

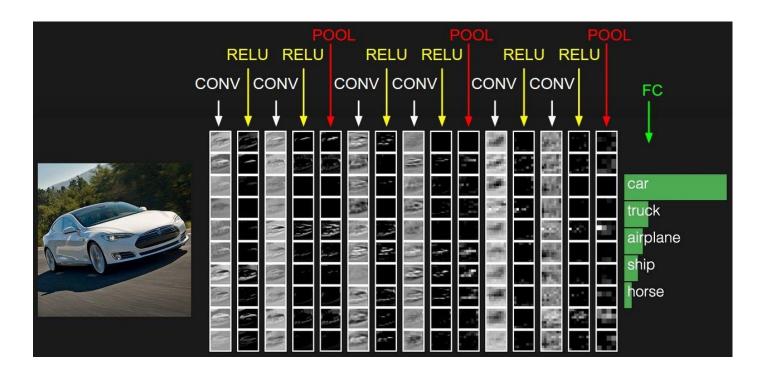


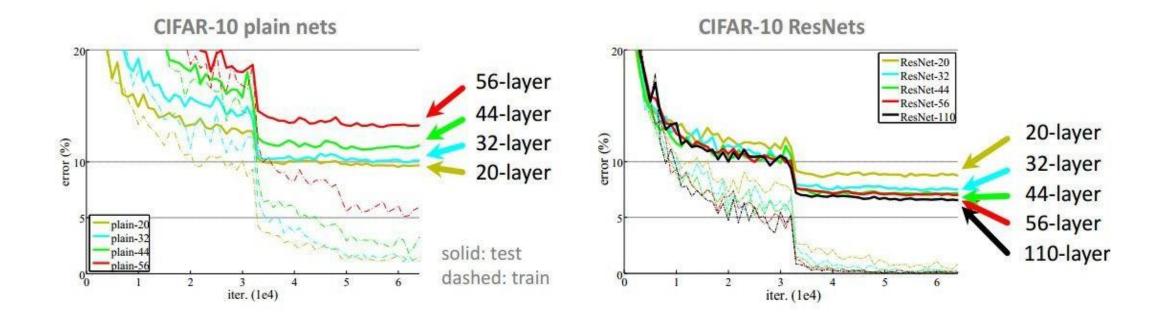
max pool with 2x2 filters and stride 2

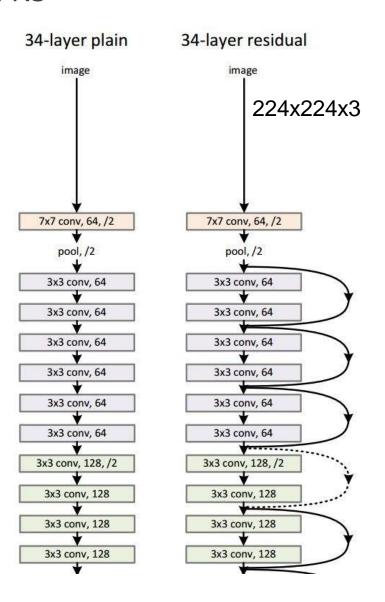
| 6 | 8 |
|---|---|
| 3 | 4 |

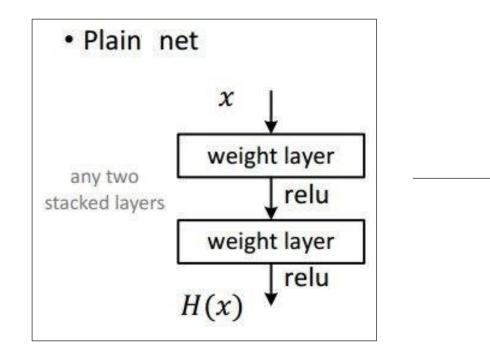
Fully Connected Layer (FC layer)

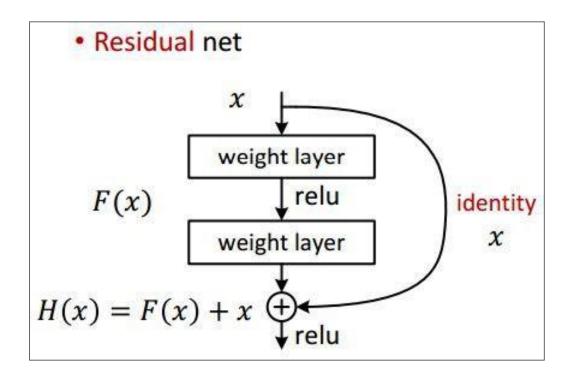
 Contains neurons that connect to the entire input volume, as in ordinary Neural Networks

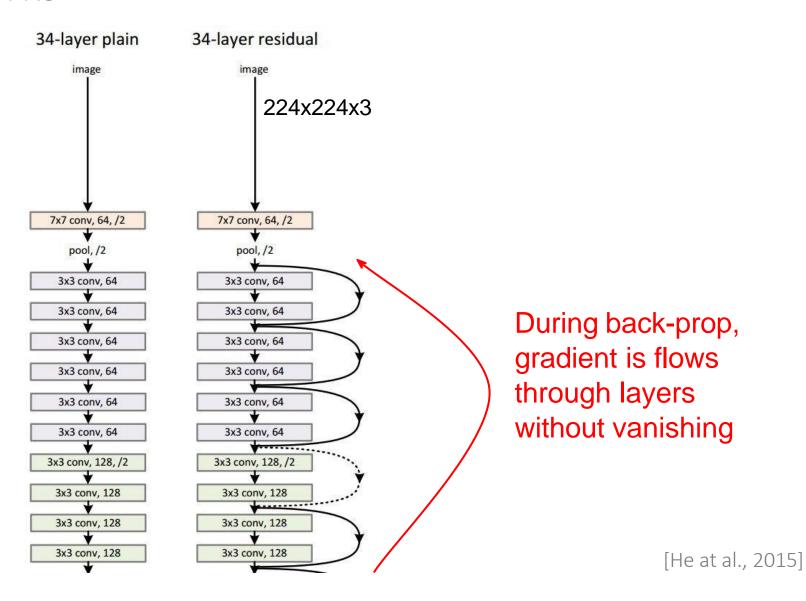


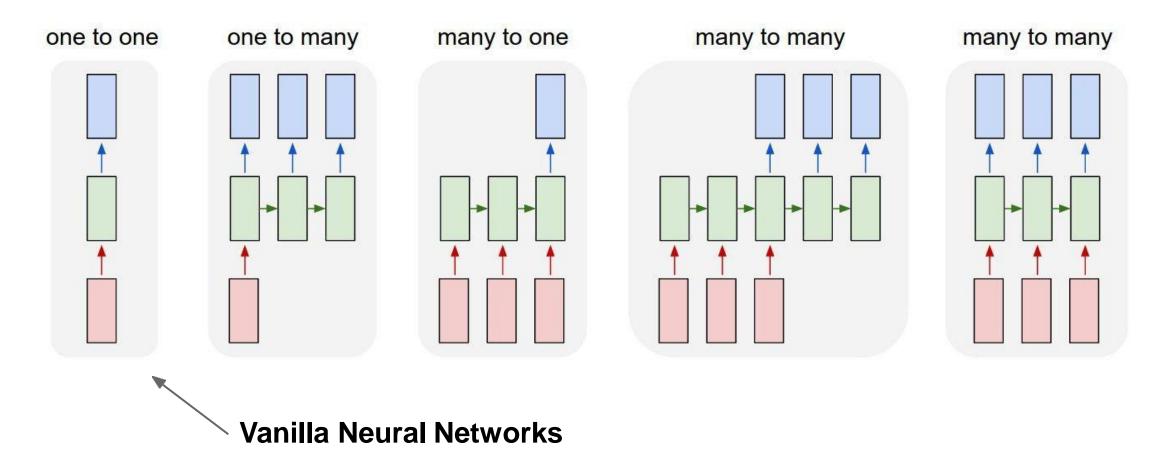


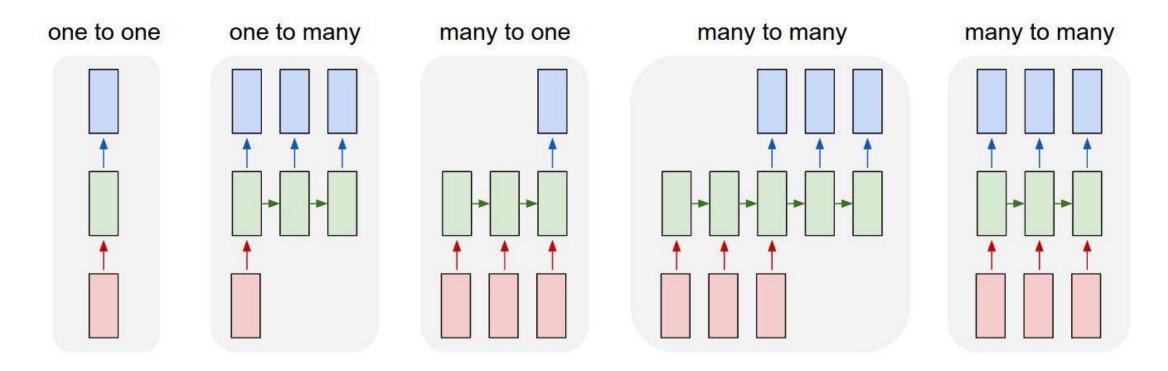




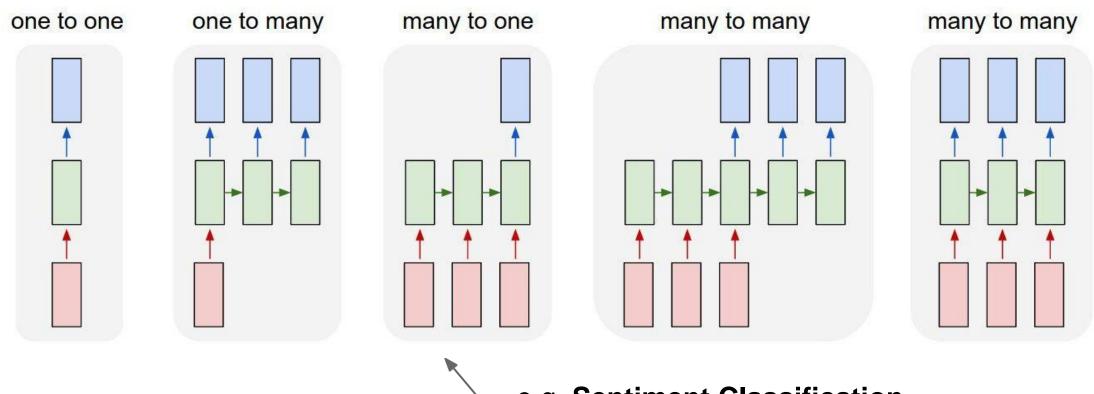




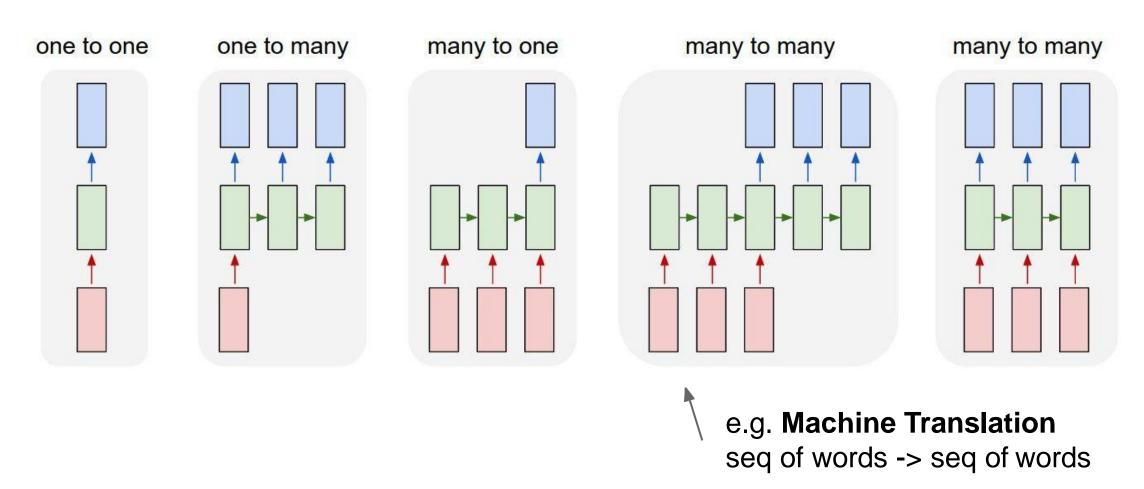


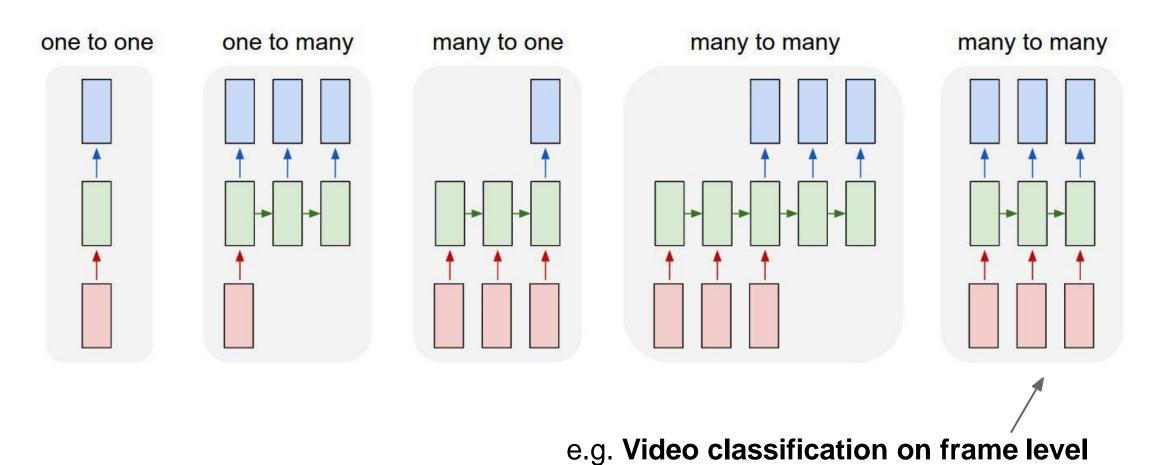


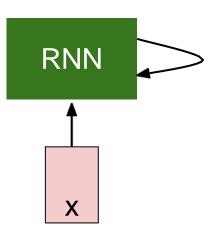
e.g. **Image Captioning** image -> sequence of words

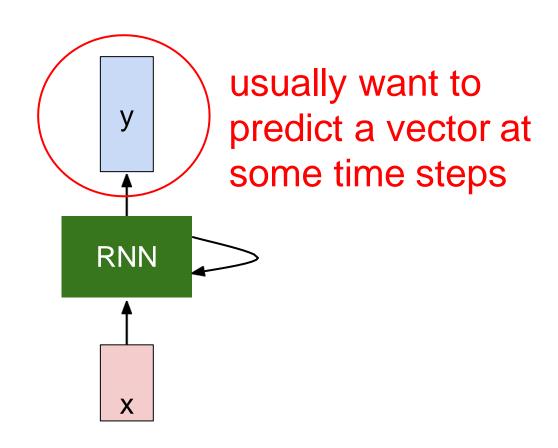


e.g. **Sentiment Classification** sequence of words -> sentiment

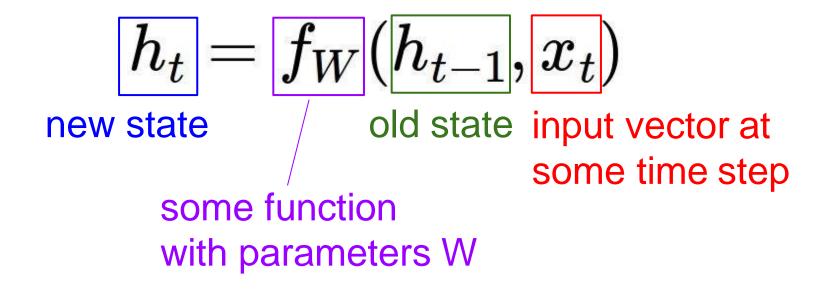


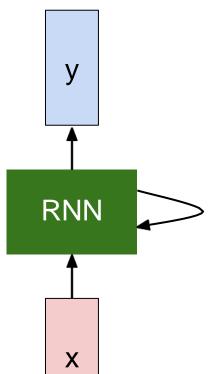






We can process a sequence of vectors **x** by applying a recurrence formula at every time step:

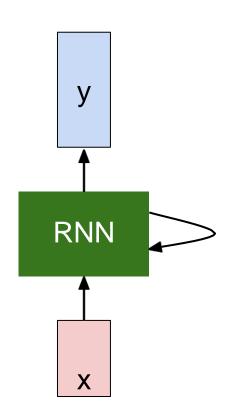




We can process a sequence of vectors **x** by applying a recurrence formula at every time step:

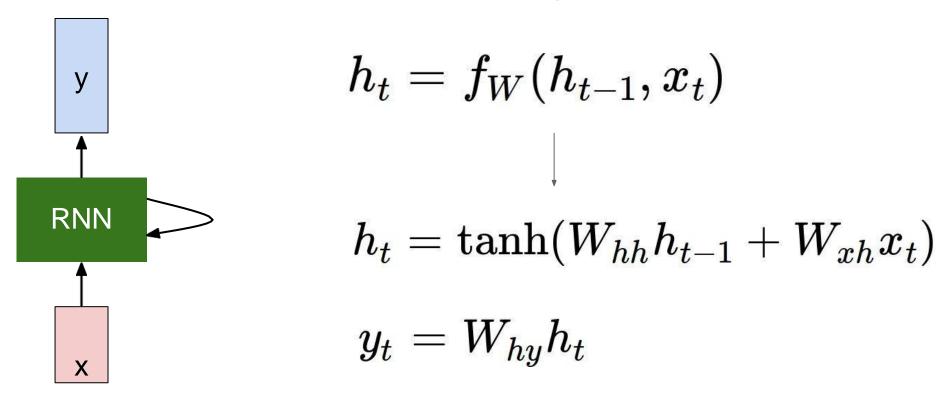
$$h_t = f_W(h_{t-1}, x_t)$$

Notice: the same function and the same set of parameters are used at every time step.



(Vanilla) Recurrent Neural Network

The state consists of a single "hidden" vector **h**:



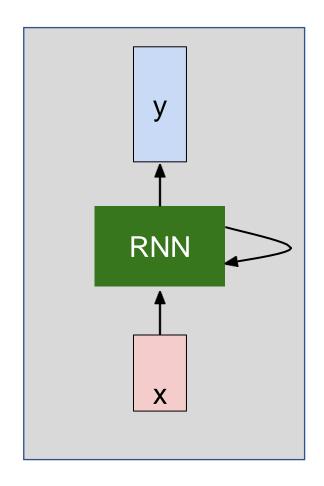
Character-level language model example

Vocabulary: [h,e,l,o]

Example training sequence:

"hello"





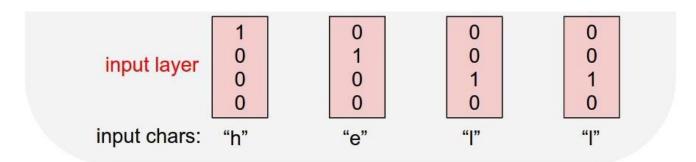
One-hot (one-of-n) encoding

Example: letters. |V| = 30

Character-level language model example

Vocabulary: [h,e,l,o]

Example training sequence: "hello"



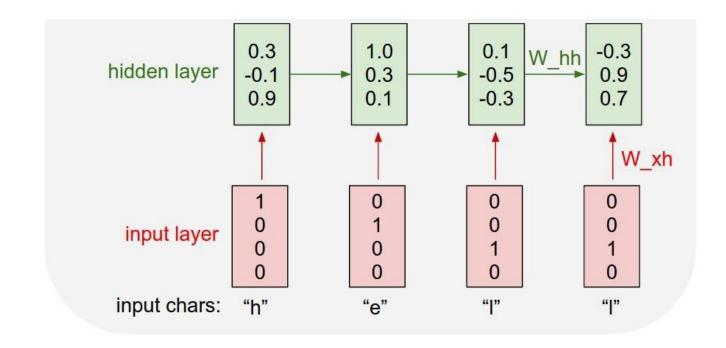
Objective:

Character-level language model example

$$h_t = anh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Vocabulary: [h,e,l,o]

Example training sequence: "hello"

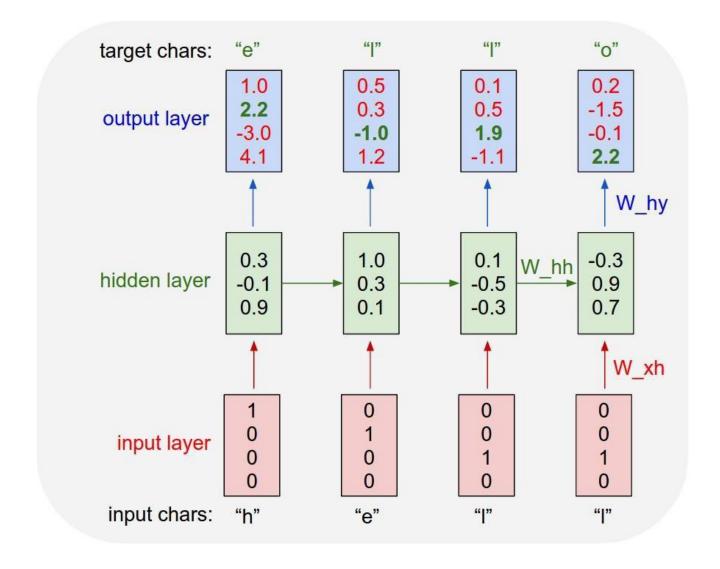


Objective:

Character-level language model example

Vocabulary: [h,e,l,o]

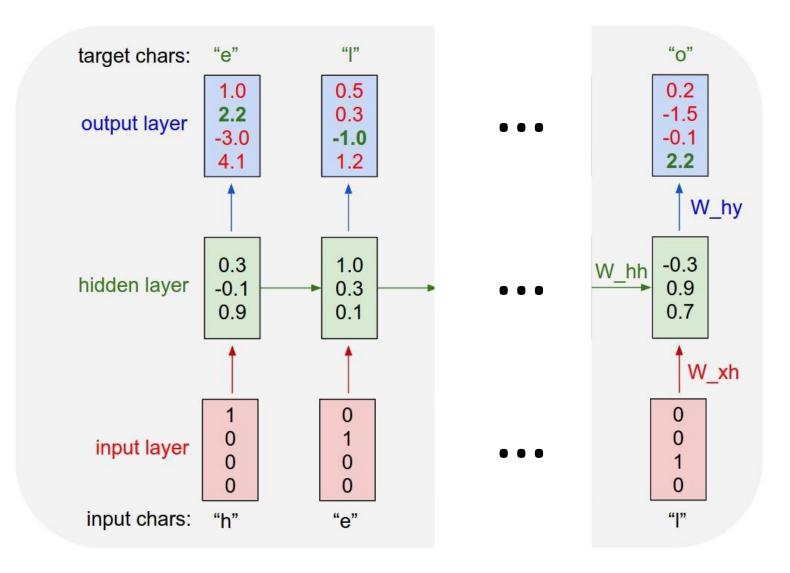
Example training sequence: "hello"



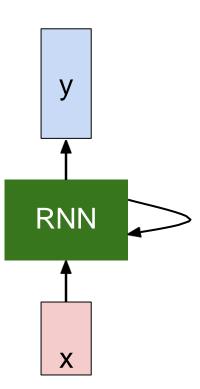
Objective:

Varying length input

Forward and backward passes are conducted on consequent subsequences iteratively



*p classe" clearex? > Products: Laser Printers. The fundamental everyday requirement for mono and colour laser printing throughbut today's offices is perfectly met with the extensive Epson laser printer range. The latest Acutuser printer range offers users exceptionally Epson Acutuser C1900. Networked compact colour laser printing for receives anot enterprise. Businesses have been denied simple and afforable colour laser printing for far too long. The traditionally high costs and poor speeds of colour lasers has left many offices looking a bit, wife yers' But on any more with the Epson Acutuser C1900. Epson Emps both colours and monotherme laser printing together at a black and withe price. more . Where to Buy Support . Epson Acutuser C1900. The fastest colour laser printing together at a black and white price. more . Where to Buy Flagon Acutuser C1900 The fastest colour laser printing at the colour laser printing together at a black and white and without an acutus of the colour laser c1900 The fastest colour laser printing together or . Where to Buy High quality resolution. Actions [187] It large puber. The colour laser c1900 The fastest colour laser printing together and the colour laser c1900 The fastest colour laser printing together. The colour laser c1900 The fastest colour laser printing together and the colour laser sections. Actions [187] It large puber. The colour laser c1900 The fastest colour laser sections are called the colour laser sections. The colour laser sections are standard. If section are feet affecting the colour laser sections are standard. If section are feet affecting the colour laser sections are standard. If section are feet affecting the colour laser sections are standard. If section are feet affecting the colour laser sections are called the colour laser and the colour laser sections. casesty, 600 steeds, expandable up to 1,000 sheets. Compatible Windows and Mac Hight speed USB and Epstantiet 01/00 Base Tx Ethernet Interfaces as standard": "Esson Acultaser C2000; Acultaser C solution. It adds crucial colour by your business, while producing high quality monothrome output at lower costs than many monochrome-only printers, and its just as easy to operate. So now there's no reason to buy two printers, because perfect monochrome and colour solutions are available in one. more: Where to Buy Support Epson Acultaer C6500 Professional high performance and copius solutions are dividable in one. Indire: Avertee to a group Apul, set Copius Professional Inging performance
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AST/0 copius laser printed Epison Acquitaer COSIO is the perfect professional Ingining solution for users who recurre decidental Apulation
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Acquitaers C1800 WF river by 2019, 600 Sheet ME Terry, 500 Sheet Cassette. Wreebes Networking Scielly. And code to your brushess Acutaser C1900 WFr with 2248, 200 Sheet K1P Tray', 500 Sheet Cassette, Wireless Networking facility. Add colour to your business with the Epson Acutaser C900 from Epson as perfect for the smaller vortices that offers among colour output as well as high Support. Epson Acutaser C4000 High performance cloud traser Theorem C700 Fig. 19 and 19 and



Sonnet 116 - Let me not ...

by William Shakespeare

Let me not to the marriage of true minds
 Admit impediments. Love is not love

Which alters when it alteration finds,
 Or bends with the remover to remove:

O no! it is an ever-fixed mark
 That looks on tempests and is never shaken;

It is the star to every wandering bark,
 Whose worth's unknown, although his height be taken.

Love's not Time's fool, though rosy lips and cheeks
 Within his bending sickle's compass come:

Love alters not with his brief hours and weeks,
 But bears it out even to the edge of doom.

If this be error and upon me proved,
 I never writ, nor no man ever loved.

at first:

tyntd-iafhatawiaoihrdemot lytdws e ,tfti, astai f ogoh eoase rrranbyne 'nhthnee e plia tklrgd t o idoe ns,smtt h ne etie h,hregtrs nigtike,aoaenns lng

train more

"Tmont thithey" fomesscerliund
Keushey. Thom here
sheulke, anmerenith ol sivh I lalterthend Bleipile shuwy fil on aseterlome
coaniogennc Phe lism thond hon at. MeiDimorotion in ther thize."

train more

Aftair fall unsuch that the hall for Prince Velzonski's that me of her hearly, and behs to so arwage fiving were to it beloge, pavu say falling misfort how, and Gogition is so overelical and ofter.

train more

"Why do what that day," replied Natasha, and wishing to himself the fact the princess, Princess Mary was easier, fed in had oftened him.

Pierre aking his soul came to the packs and drove up his father-in-law women.

PANDARUS:

Alas, I think he shall be come approached and the day When little srain would be attain'd into being never fed, And who is but a chain and subjects of his death, I should not sleep.

Second Senator:

They are away this miseries, produced upon my soul, Breaking and strongly should be buried, when I perish The earth and thoughts of many states.

DUKE VINCENTIO:

Well, your wit is in the care of side and that.

Second Lord:

They would be ruled after this chamber, and my fair nues begun out of the fact, to be conveyed, Whose noble souls I'll have the heart of the wars.

Clown:

Come, sir, I will make did behold your worship.

VIOLA:

I'll drink it.

VIOLA:

Why, Salisbury must find his flesh and thought
That which I am not aps, not a man and in fire,
To show the reining of the raven and the wars
To grace my hand reproach within, and not a fair are hand,
That Caesar and my goodly father's world;
When I was heaven of presence and our fleets,
We spare with hours, but cut thy council I am great,
Murdered and by thy master's ready there
My power to give thee but so much as hell:
Some service in the noble bondman here,
Would show him to her wine.

KING LEAR:

O, if you were a feeble sight, the courtesy of your law, Your sight and several breath, will wear the gods With his heads, and my hands are wonder'd at the deeds, So drop upon your lordship's head, and your opinion Shall be against your honour.

RNN:

$$h_t^l = \tanh W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

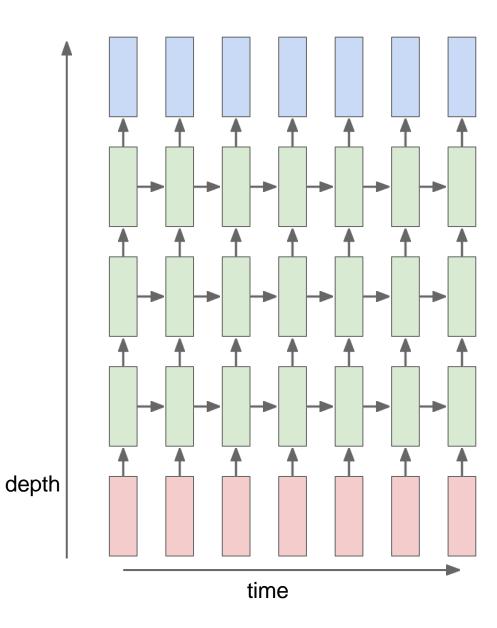
$$h \in \mathbb{R}^n \qquad W^l \quad [n \times 2n]$$

A generalization of RNN. At I=1:

- $h_t^{l-1} = X_t$
- $\bullet \quad W^{I} = [W_{xh} W_{hh}]$

It is equivalent to:

$$h_t = anh(W_{hh}h_{t-1} + W_{xh}x_t)$$



RNN:

$$h_t^l = \tanh W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$$h \in \mathbb{R}^n \qquad W^l \quad [n \times 2n]$$

LSTM:

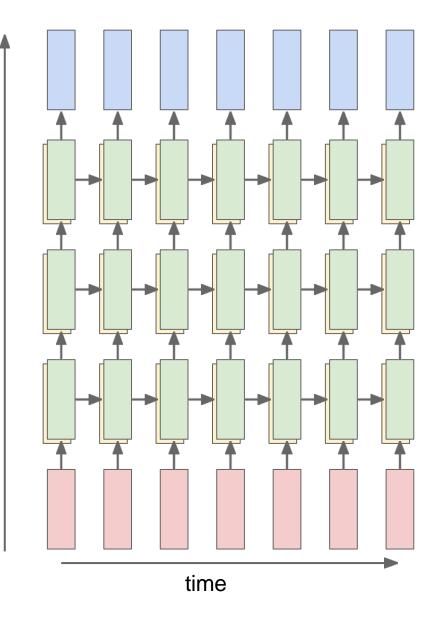
$$W^l [4n \times 2n]$$

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \operatorname{sigm} \\ \operatorname{sigm} \\ \operatorname{sigm} \\ \tanh \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

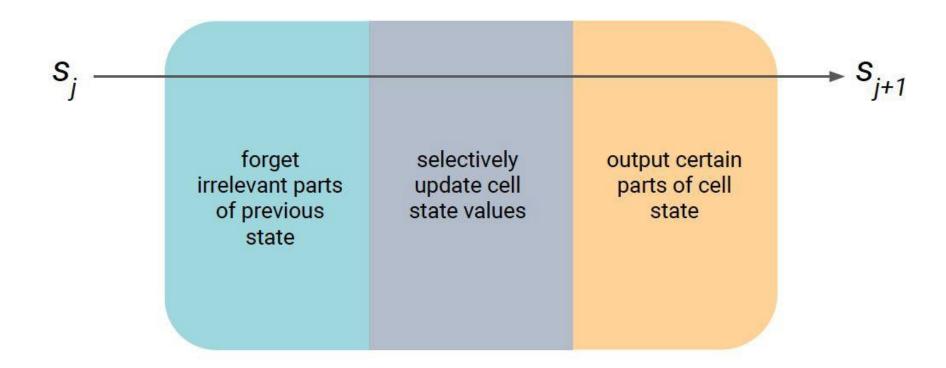
$$c_t^l = f \odot c_{t-1}^l + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$

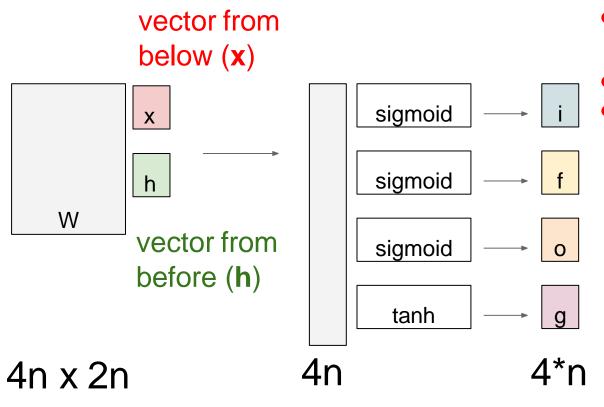
depth



LSTM - main idea



Long Short Term Memory (LSTM) [Hochreiter et al., 1997]



- c: cell state
- h: hidden state (cell output)
- i: input gate, weight of acquiring new information
- f: forget gate, weight of remembering old information
- g: transformed input ([-1,+1])
- o: output gate, decides values to be activated based on current memory

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \operatorname{sigm} \\ \operatorname{sigm} \\ \operatorname{sigm} \\ \tanh \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$$c_t^l = f \odot c_{t-1}^l + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$

Long Short Term Memory (LSTM) [Hochreiter et al., 1997]

vector from below (x) sigmoid Χ sigmoid h W vector from sigmoid before (h) tanh 4*n 4n 4n x 2n

f decides the degree of preservation for cell state, by scaling it with a number in [0,1]

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \operatorname{sigm} \\ \operatorname{sigm} \\ \operatorname{sigm} \\ \tanh \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$$c_t^l = \underbrace{f \odot c_{t-1}^l}_{l} + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$

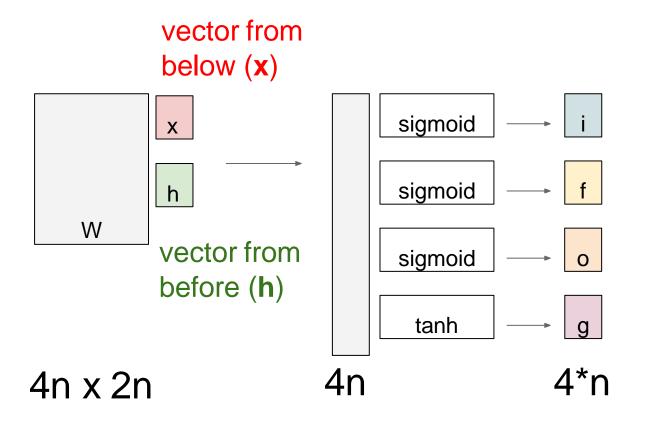
vector from below (x) sigmoid Χ sigmoid h W vector from sigmoid before (h) tanh 4*n 4n 4n x 2n

g is a transformation of input / hidden state

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \operatorname{sigm} \\ \operatorname{sigm} \\ \operatorname{sigm} \\ \tanh \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$$c_t^l = f \odot c_{t-1}^l + i \odot \boxed{g}$$

$$h_t^l = o \odot \tanh(c_t^l)$$



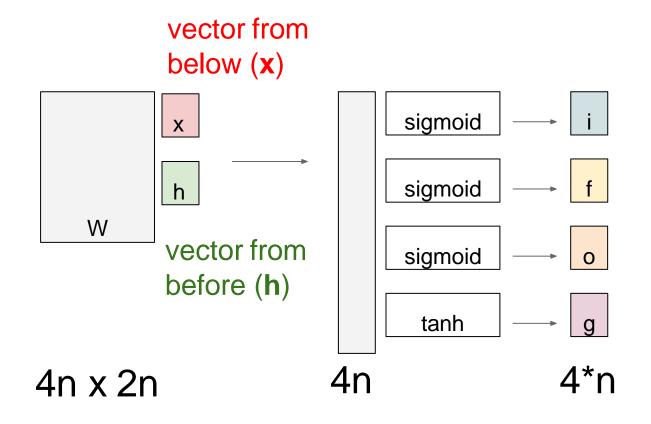
Add *g* into the *cell state*, weighted by *i* (weight of acquiring new information)

Alternative interpretation: i*g decouples the "influence of g" and "g itself".

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \operatorname{sigm} \\ \operatorname{sigm} \\ \operatorname{sigm} \\ \tanh \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$$c_t^l = f \odot c_{t-1}^l + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$



New hidden state is a scaled version of tanh(cell state).

o: output gate, decides values to be activated based on current memory

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \operatorname{sigm} \\ \operatorname{sigm} \\ \operatorname{sigm} \\ \tanh \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$$c_t^l = f \odot c_{t-1}^l + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$

vector from below (x) sigmoid Χ sigmoid h W vector from sigmoid before (h) tanh 4*n 4n 4n x 2n

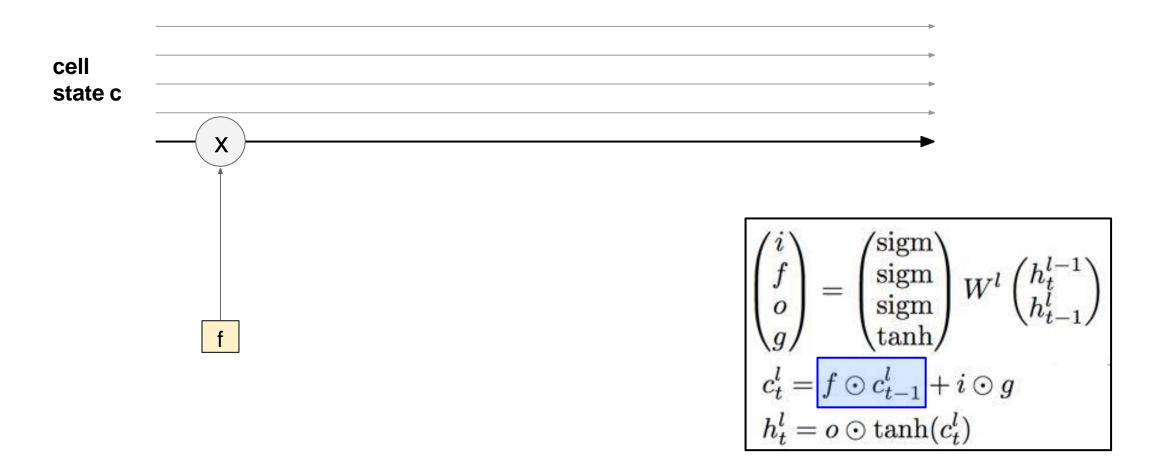
Q: Why tanh?

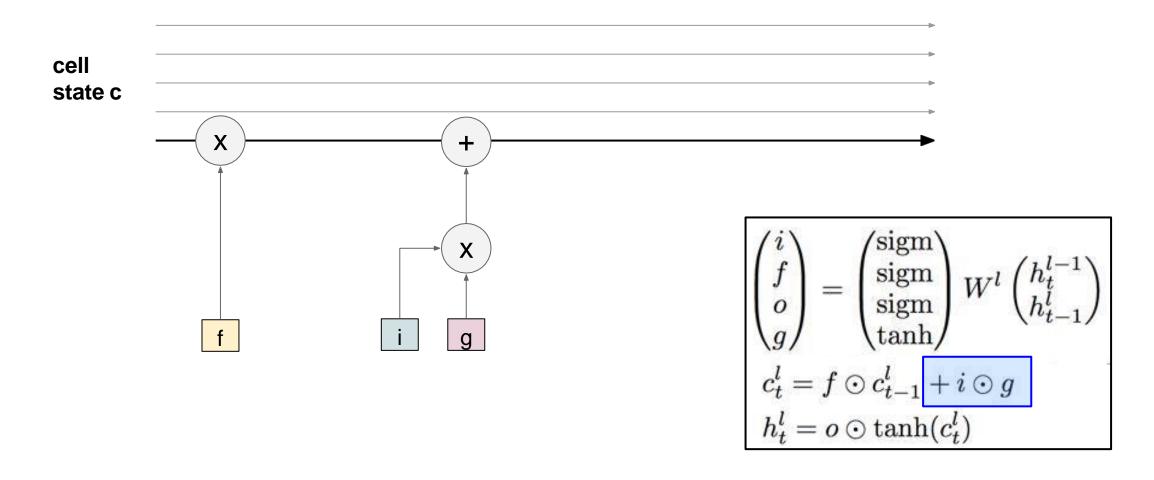
A: Not very crucial, sometimes not used

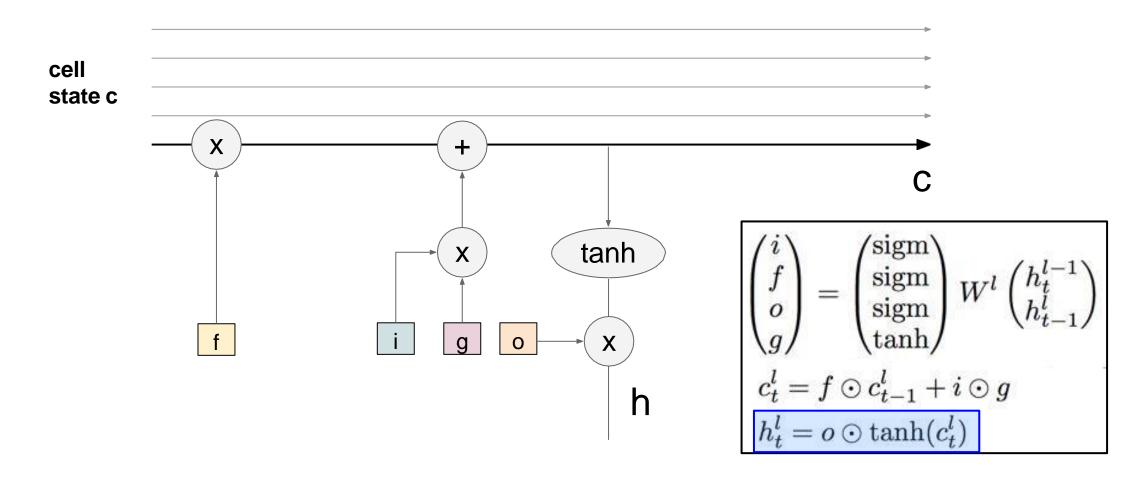
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \operatorname{sigm} \\ \operatorname{sigm} \\ \operatorname{sigm} \\ \tanh \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$$c_t^l = f \odot c_{t-1}^l + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$



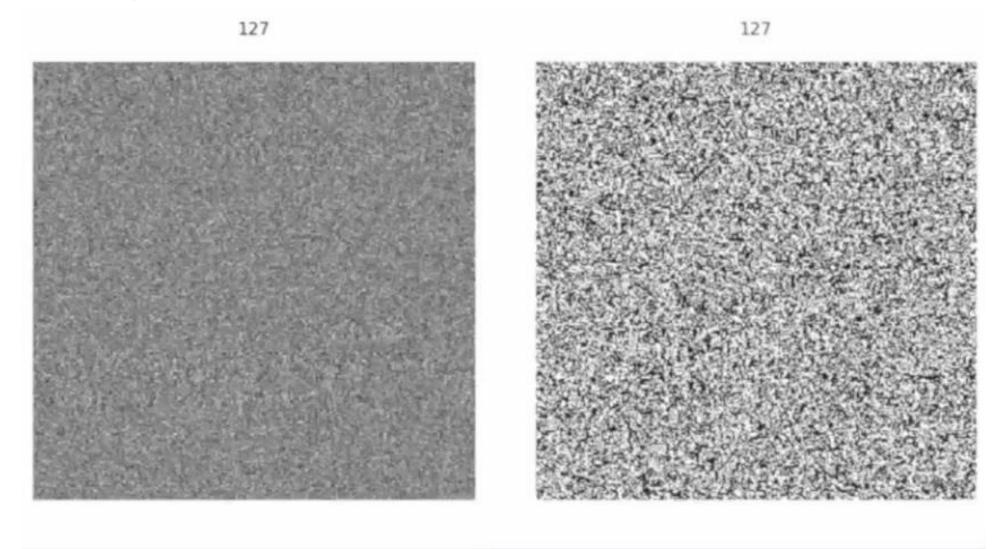




Long Short Term Memory (LSTM) higher layer, or [Hochreiter et al., 1997] prediction cell state c tanh

Understanding gradient flow dynamics

Backprop signal



Understanding gradient flow dynamics

Backprop signal video: http://imgur.com/gallery/vaNahKE

In RNN, the gradient vanishes much more quickly as we backprop from the last time step towards the first one

Therefore, RNN here cannot learn long time dependencies

Understanding gradient flow dynamics RNN without any inputs

```
# dimensionality of hidden state
H = 5
T = 50 # number of time steps
Whh = np.random.randn(H,H)
# forward pass of an RNN (ignoring inputs x)
hs = \{\}
ss = \{\}
hs[-1] = np.random.randn(H)
for t in xrange(T):
    ss[t] = np.dot(Whh, hs[t-1])
    hs[t] = np.maximum(0, ss[t])
# backward pass of the RNN
dhs = \{\}
dss = \{\}
dhs[T-1] = np.random.randn(H) # start off the chain with random gradient
for t in reversed(xrange(T)):
    dss[t] = (hs[t] > 0) * dhs[t] # backprop through the nonlinearity
    dhs[t-1] = np.dot(Whh.T, dss[t]) # backprop into previous hidden state
```

Understanding gradient flow dynamics RNN without any inputs

```
# dimensionality of hidden state
H = 5
T = 50 # number of time steps
Whh = np.random.randn(H,H)
                                                      Back-propagation signal is repeatedly multiplied by Whh.
# forward pass of an RNN (ignoring inputs x)
hs = \{\}
ss = \{\}
hs[-1] = np.random.randn(H)
for t in xrange(T):
    ss[t] = np.dot(Whh, hs[t-1])
    hs[t] = np.maximum(0, ss[t])
# backward pass of the RNN
dhs = \{\}
dss = \{\}
dhs[T-1] = np.random.randn(H) # start off/the chain with random gradient
for t in reversed(xrange(T)):
    dss[t] = (hs[t] > 0) * dhs[t] # backprop through the nonlinearity
    dhs[t-1] = np.dot(Whh.T, dss[t]) # backprop into previous hidden state
```

[On the difficulty of training Recurrent Neural Networks, Pascanu et al., 2013]

Understanding gradient flow dynamics RNN without any inputs

```
# dimensionality of hidden state
H = 5
T = 50 # number of time steps
                                                    if the largest eigenvalue is < 1, gradient will vanish
Whh = np.random.randn(H,H)
                                                    if the largest eigenvalue is > 1, gradient will explode
# forward pass of an RNN (ignoring inputs x)
hs = \{\}
ss = \{\}
hs[-1] = np.random.randn(H)
for t in xrange(T):
   ss[t] = np.dot(Whh, hs[t-1])
                                                       can control vanishing with LSTM
   hs[t] = np.maximum(0, ss[t])
                                                       can control exploding with gradient clipping
# backward pass of the RNN
dhs = \{\}
dss = \{\}
dhs[T-1] = np.random.randn(H) # start off the chain with random gradient
for t in reversed(xrange(T)):
   dss[t] = (hs[t] > 0) * dhs[t] # backprop through the nonlinearity
   dhs[t-1] = np.dot(Whh.T, dss[t]) # backprop into previous hidden state
```

[On the difficulty of training Recurrent Neural Networks, Pascanu et al., 2013]

Vanishing gradient problem

An example how vanishing gradient problem can affect RNNs:

"In France, I had a great time and I learnt some of the ____ language."

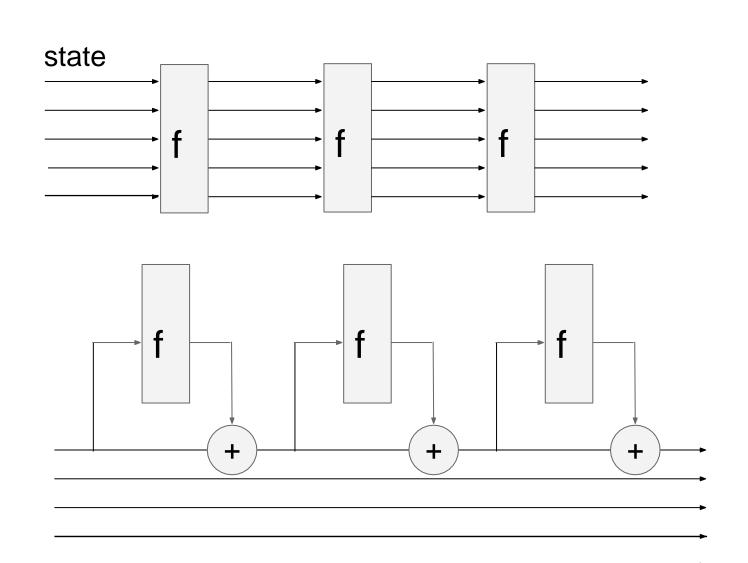
our parameters are not trained to capture long-term dependencies, so the word we predict will mostly depend on the previous few words, not much earlier ones

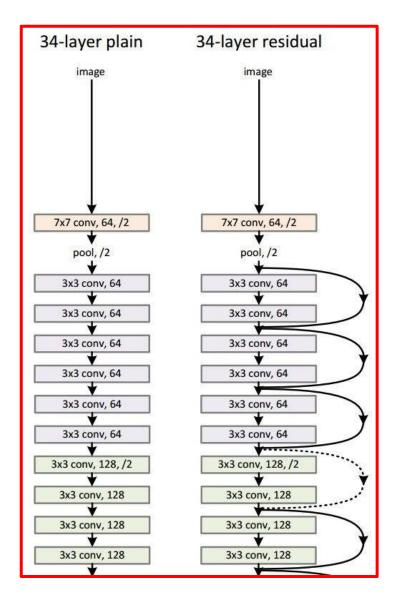
RNN

More prone to the vanishing gradient problem



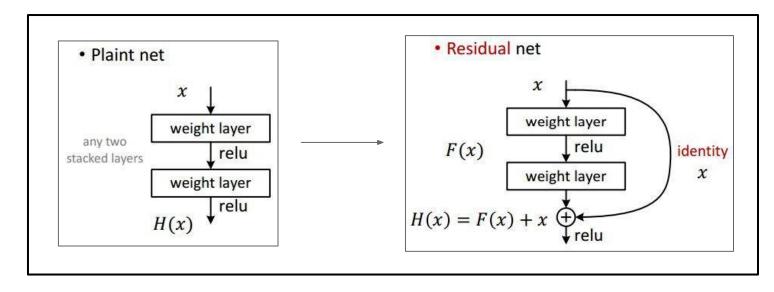
(ignoring forget gates)





Recall: "PlainNets" vs. ResNets

ResNet is to PlainNet what LSTM is to RNN, kind of.



Vanishing gradient problem summary

To address this problem, use

- better activation function (eg, ReLU)
- proper initialization (W=Identity, bias=zeros) to prevent W from shrinking the gradients
- replace RNN cells with LSTM or other gated cells (LSTM variants) to control what information is passed through