1. Consider a relation $R$ with five attributes $ABCDE$. You are given the following dependencies: $A \rightarrow B$, $BC \rightarrow E$, $ED \rightarrow A$.

1. List all keys for $R$.
2. Is $R$ in 3NF?
3. Is $R$ in BCNF?

2. Consider the following relation.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$y_1$</td>
<td>$z_1$</td>
</tr>
<tr>
<td>$x_1$</td>
<td>$y_1$</td>
<td>$z_2$</td>
</tr>
<tr>
<td>$x_2$</td>
<td>$y_1$</td>
<td>$z_1$</td>
</tr>
<tr>
<td>$x_2$</td>
<td>$y_1$</td>
<td>$z_3$</td>
</tr>
</tbody>
</table>

1. List all the functional dependencies that this relation instance satisfies.
2. Assume that the value of attribute $Z$ of the last record in the relation is changed from $z_3$ to $z_2$. Now list all the functional dependencies that this relation instance satisfies.

3. Consider the following collection of relations and dependencies. Assume that each relation is obtained through decomposition from a relation with attributes $ABCDEFGHI$ and that all the known dependencies over relation $ABCDEFGHI$ are listed for each question. (The questions are independent of each other, obviously, since the given dependencies over $ABCDEFGHI$ are different.) For each (sub)relation: (a) State the strongest normal form that the relation is in. (b) If it is not in BCNF, decompose it into a collection of BCNF relations.

1. $R_1(A, B, C, D, E), A \rightarrow B, C \rightarrow D$
2. $R_2(A, B, F), AC \rightarrow E, B \rightarrow F$
3. $R_3(A, D), D \rightarrow G, G \rightarrow H$
4. $R_4(D, C, H, G), A \rightarrow I, I \rightarrow A$
5. $R_5(A, I, C, E)$
4. Suppose that we have the following three tuples in a legal instance of a relation schema \( S \) with three attributes \( ABC \) (listed in order): \((1,2,3), (4,2,3), \) and \((5,3,3)\).

1. Which of the following dependencies can you infer does not hold over schema \( S \)?
   
   (a) \( A \to B \), (b) \( BC \to A \), (c) \( B \to C \)

2. Can you identify any dependencies that hold over \( S \)?

5. Suppose you are given a relation \( R \) with four attributes \( ABCD \). For each of the following sets of FDs, assuming those are the only dependencies that hold for \( R \), do the following: (a) Identify the candidate key(s) for \( R \). (b) Identify the best normal form that \( R \) satisfies (1NF, 2NF, 3NF, or BCNF). (c) If \( R \) is not in BCNF, decompose it into a set of BCNF relations that preserve the dependencies.

   1. \( C \to D, C \to A, B \to C \)
   2. \( B \to C, D \to A \)
   3. \( ABC \to D, D \to A \)
   4. \( A \to B, BC \to D, A \to C \)
   5. \( AB \to C, AB \to D, C \to A, D \to B \)

6. Suppose you are given a relation \( R(A, B, C, D) \). For each of the following sets of FDs, assuming they are the only dependencies that hold for \( R \), do the following: (a) Identify the candidate key(s) for \( R \). (b) State whether or not the proposed decomposition of \( R \) into smaller relations is a good decomposition and briefly explain why or why not.

   1. \( B \to C, D \to A \); decompose into \( BC \) and \( AD \).
   2. \( AB \to C, C \to A, C \to D \); decompose into \( ACD \) and \( BC \).
   3. \( A \to BC, C \to AD \); decompose into \( ABC \) and \( AD \).
   4. \( A \to B, B \to C, C \to D \); decompose into \( AB \) and \( ACD \).
   5. \( A \to B, B \to C, C \to D \); decompose into \( AB, AD \) and \( CD \).