
Cameras

CS 554 – Computer Vision

Pinar Duygulu

Bilkent University

Cameras

Many types of imaging devices:
animal eyes, video cameras, radio telescopes

First models of camera obscura (dark chamber)
invented in 16th century
does not have lenses
use a pinhole to focus light rays onto a wall or translucent plate

They are replaced with lenses in 1550

Image formation on the backplate of a photographic camera

Modern photographic or digital camera is essentially a camera obscura capable of recording the amount of light striking every small area of its backplane

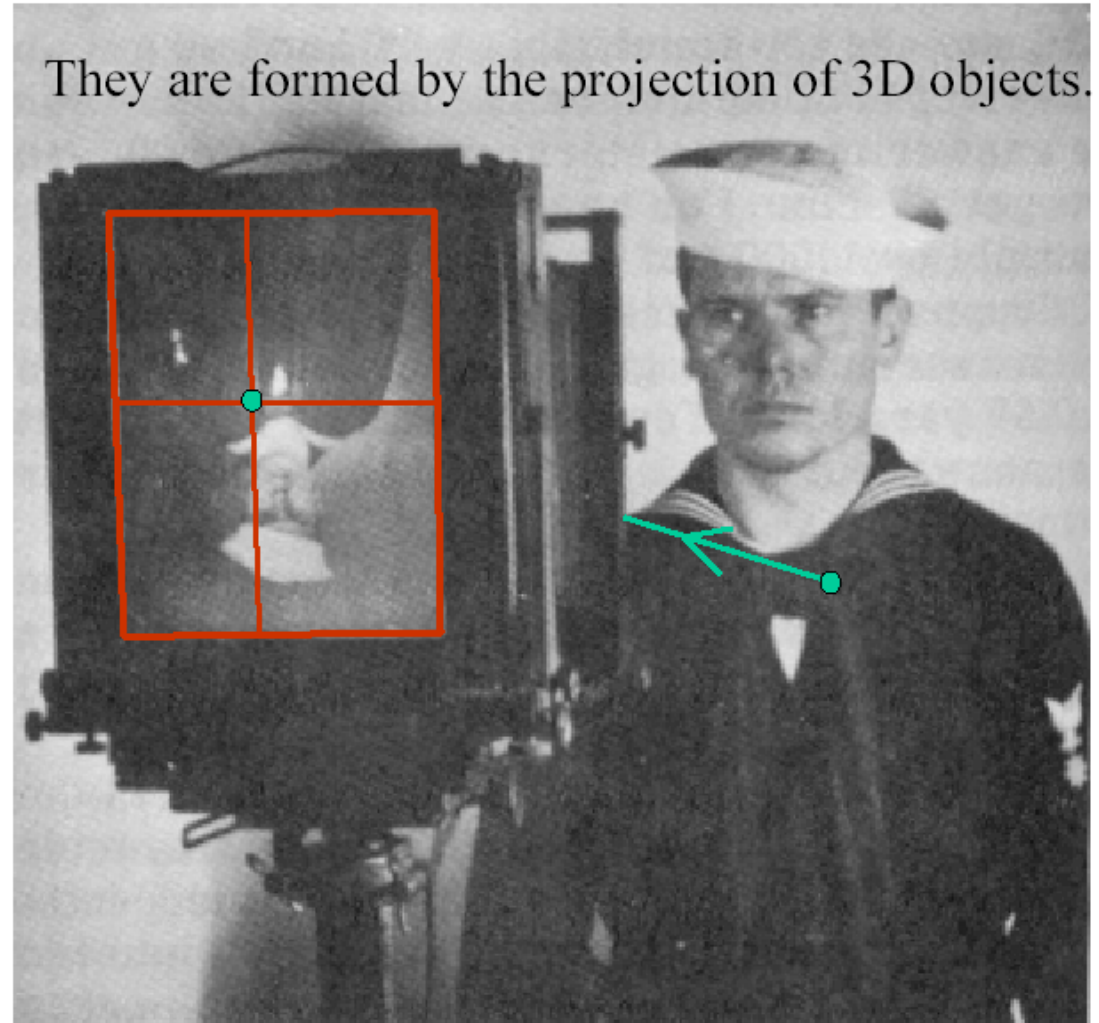


Figure from US Navy Manual of Basic Optics and Optical Instruments, prepared by Bureau of Naval Personnel. Reprinted by Dover Publications, Inc., 1969.

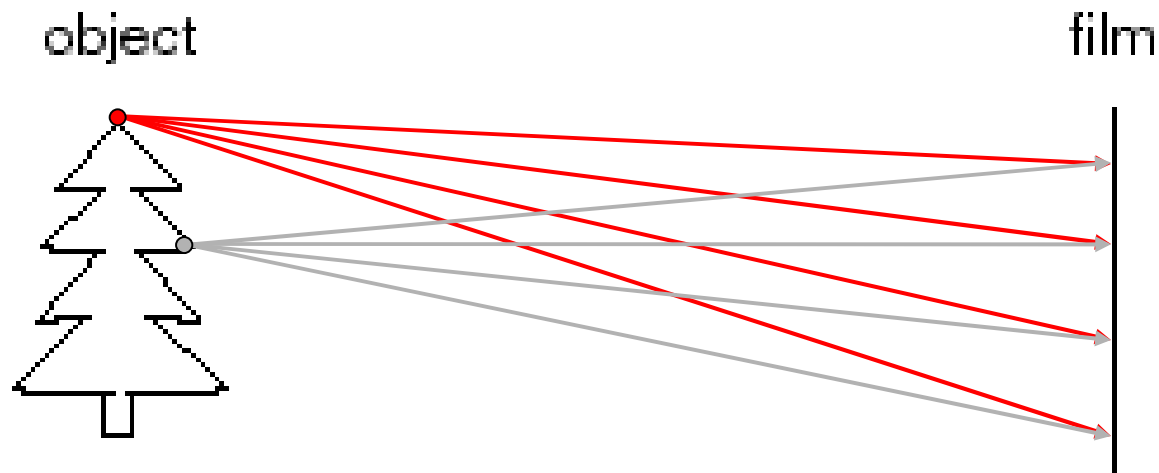
First photograph due to Niepce

- It was known since Middle ages that certain silver salts rapidly darken under the action of sunlight
- In 1816 Niepce obtained the first true photograph by exposing paper treated with silver chloride to the light rays striking the image plane of a camera obscura, then fixing the picture with nitric acid



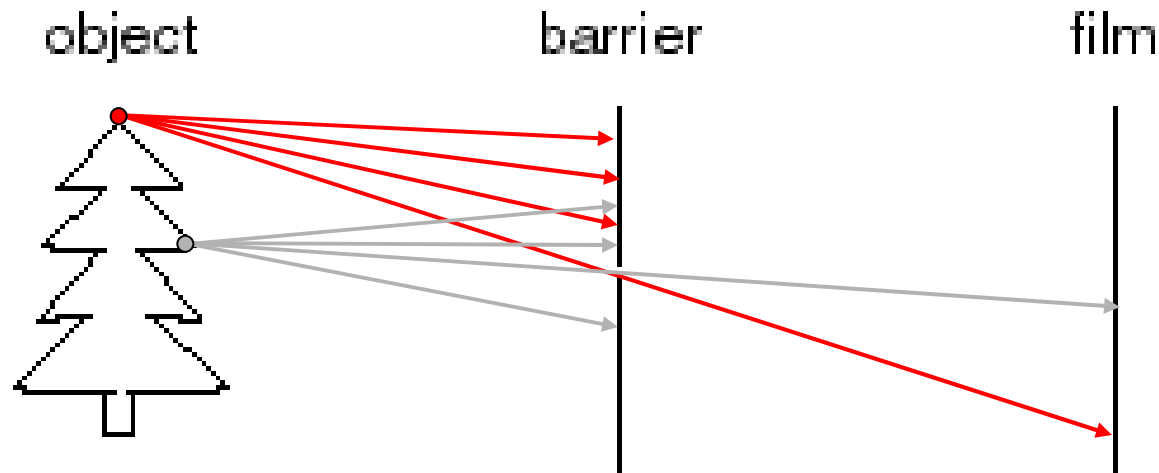
Figure 1.16 The first photograph on record, *la table servie*, obtained Nicéphore Niepce in 1822. *Collection Harlinge-Viollet.*

Image formation



- Let's design a camera
 - Idea 1: put a piece of film in front of an object
 - Do we get a reasonable image?

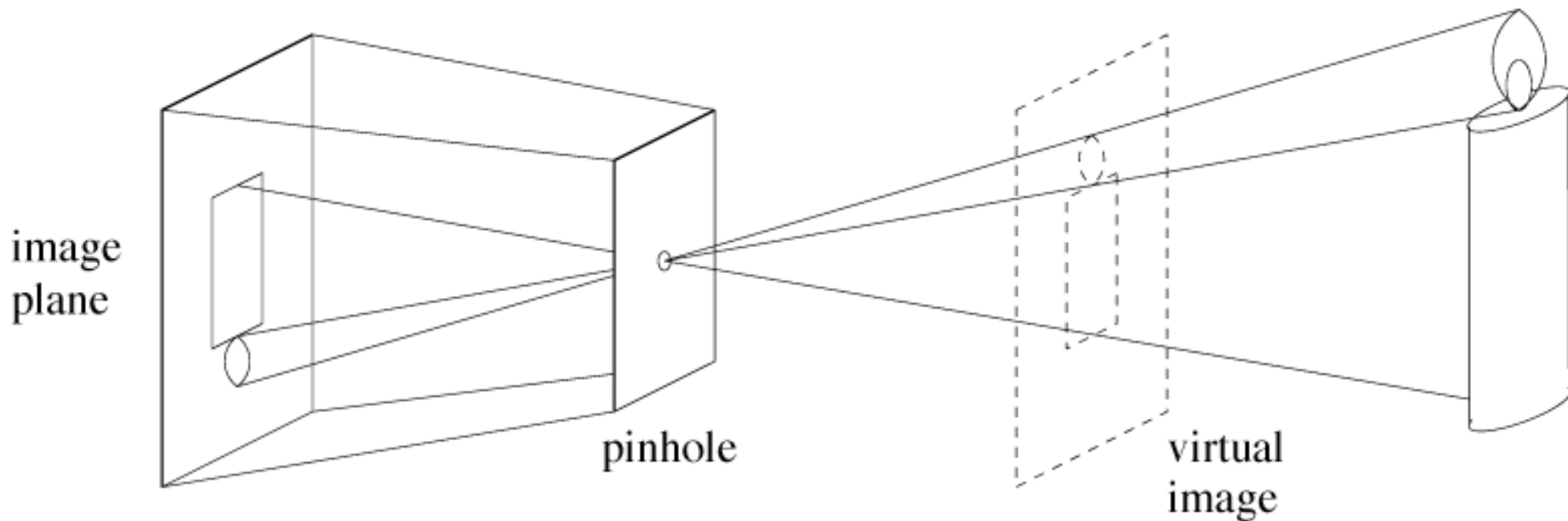
Pinhole Cameras



- Add a barrier to block off most of the rays
 - This reduces blurring
 - The opening known as the **aperture**

Pinhole cameras

- box with a small hole in it
- Replace the opposite side with a translucent plate
- The image is formed by light rays issued from the scene facing the box



Adapted from David Forsyth, UC Berkeley

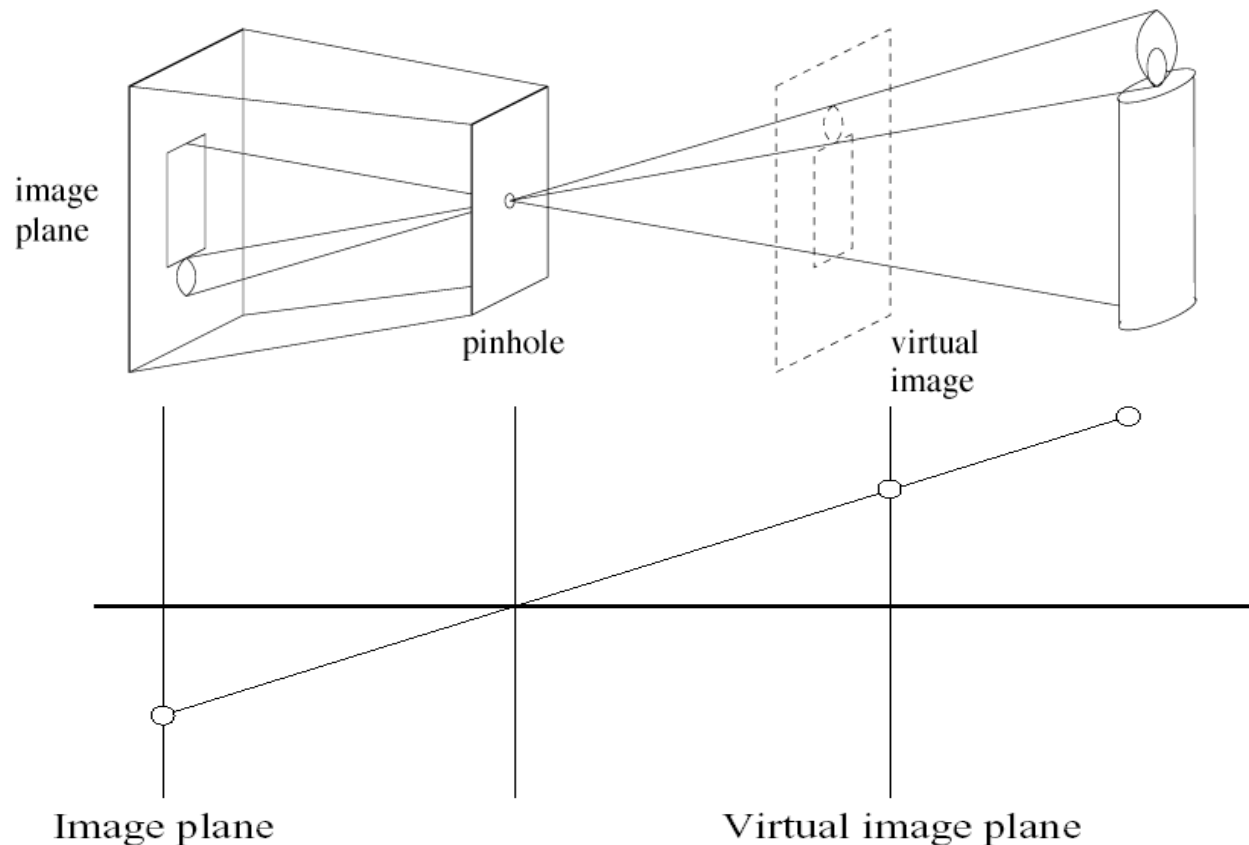
Pinhole cameras

- Abstract camera model - work in practice
- Pinhole perspective (central perspective) - first proposed by Brunelleschi at the beginning of 15th century
- Despite its simplicity it provides an acceptable approximation of the imaging process

Pinhole cameras

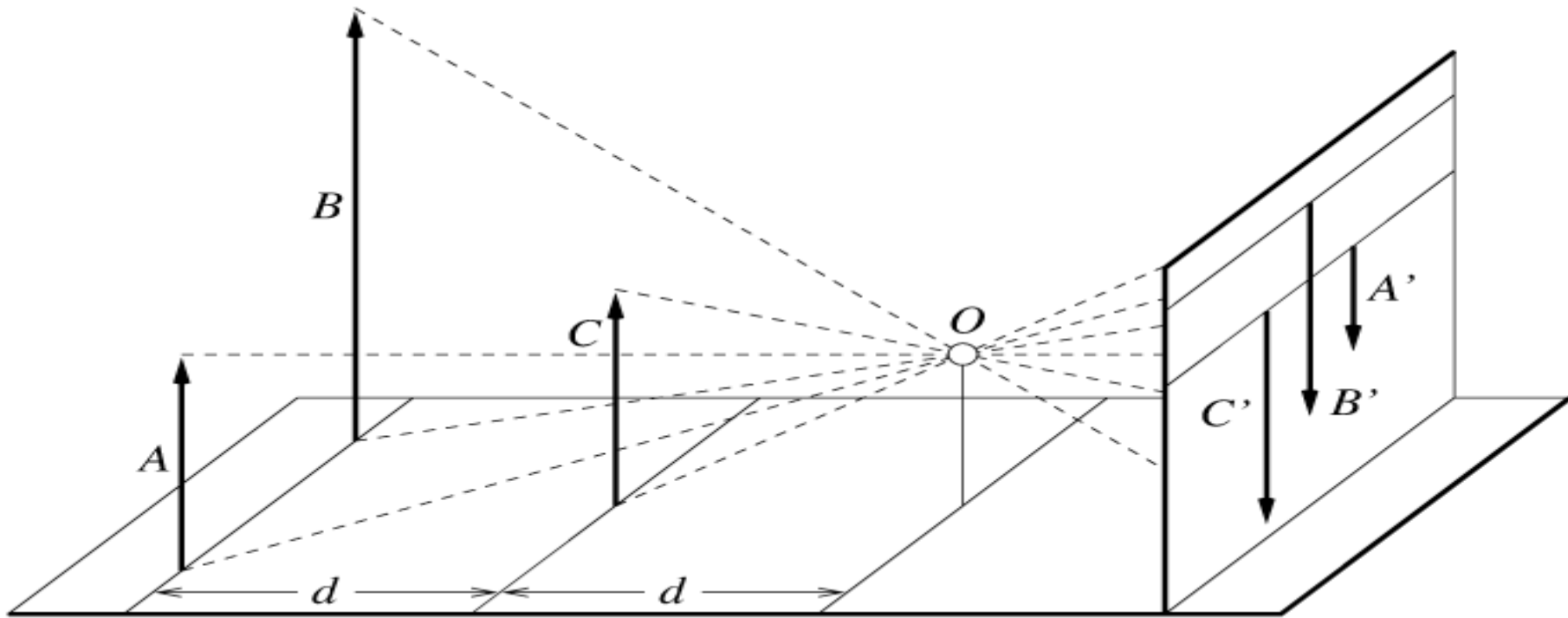
Perspective projection creates inverted images

Equivalent Model with Virtual Image Plane – plane lying in front of the pinhole at the same distance from it as the actual image plane

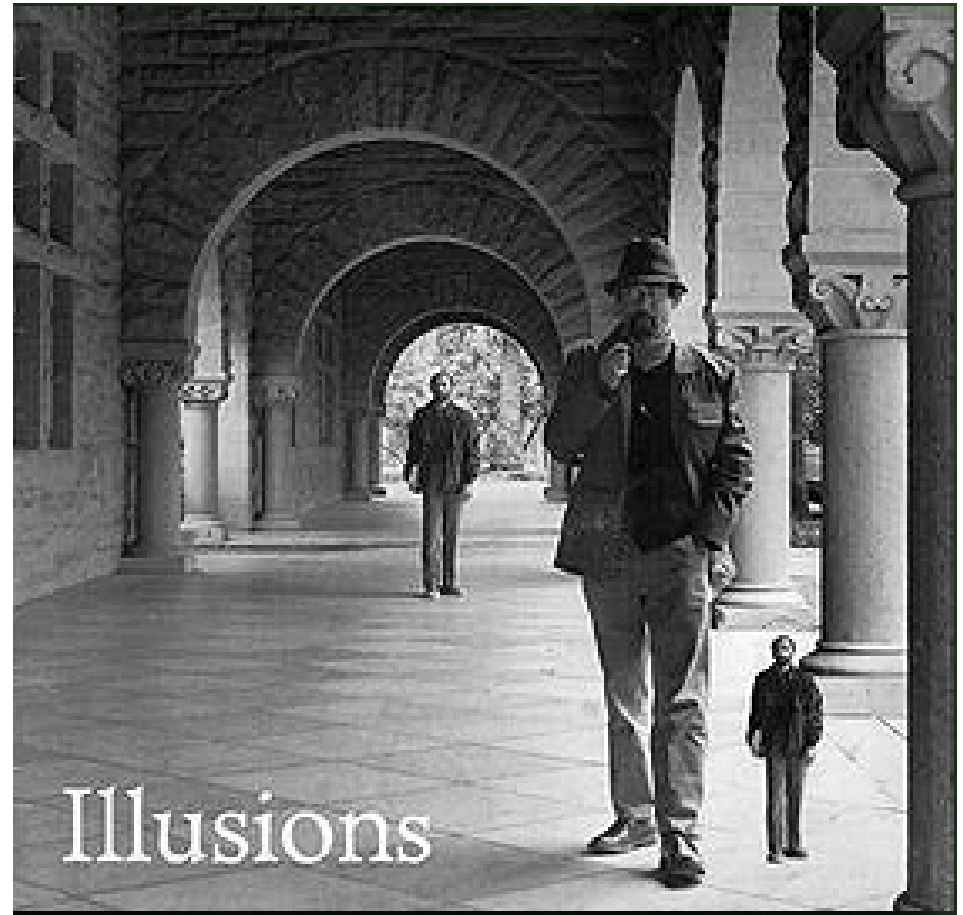
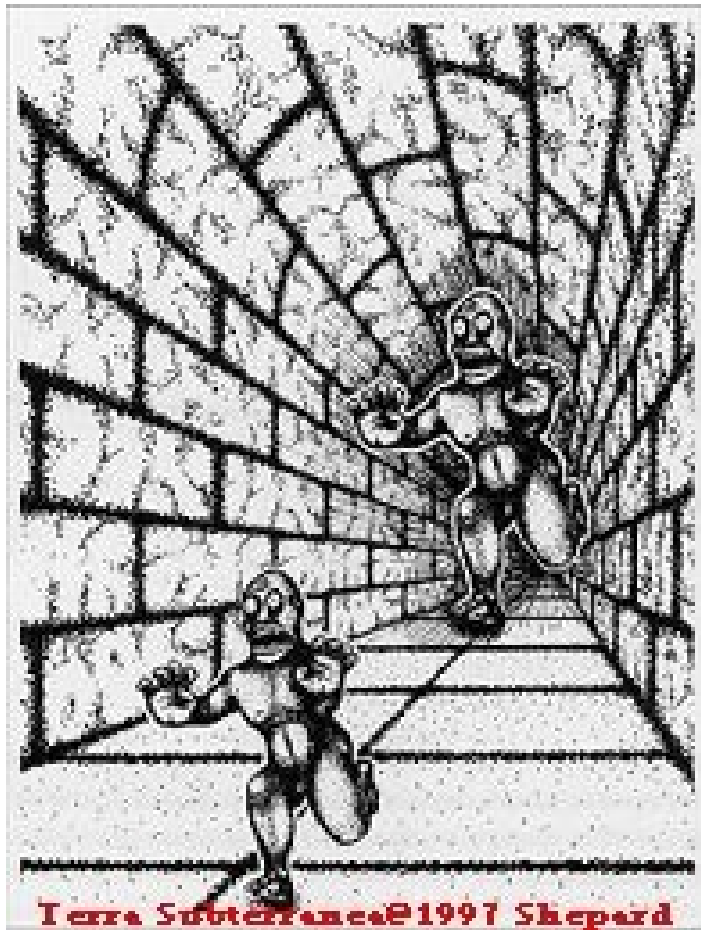


Distant objects are smaller

- Apparent size of the objects depends on their distance
- B' and C' have same length although A and C are really half size of B



Adapted from David Forsyth, UC Berkeley

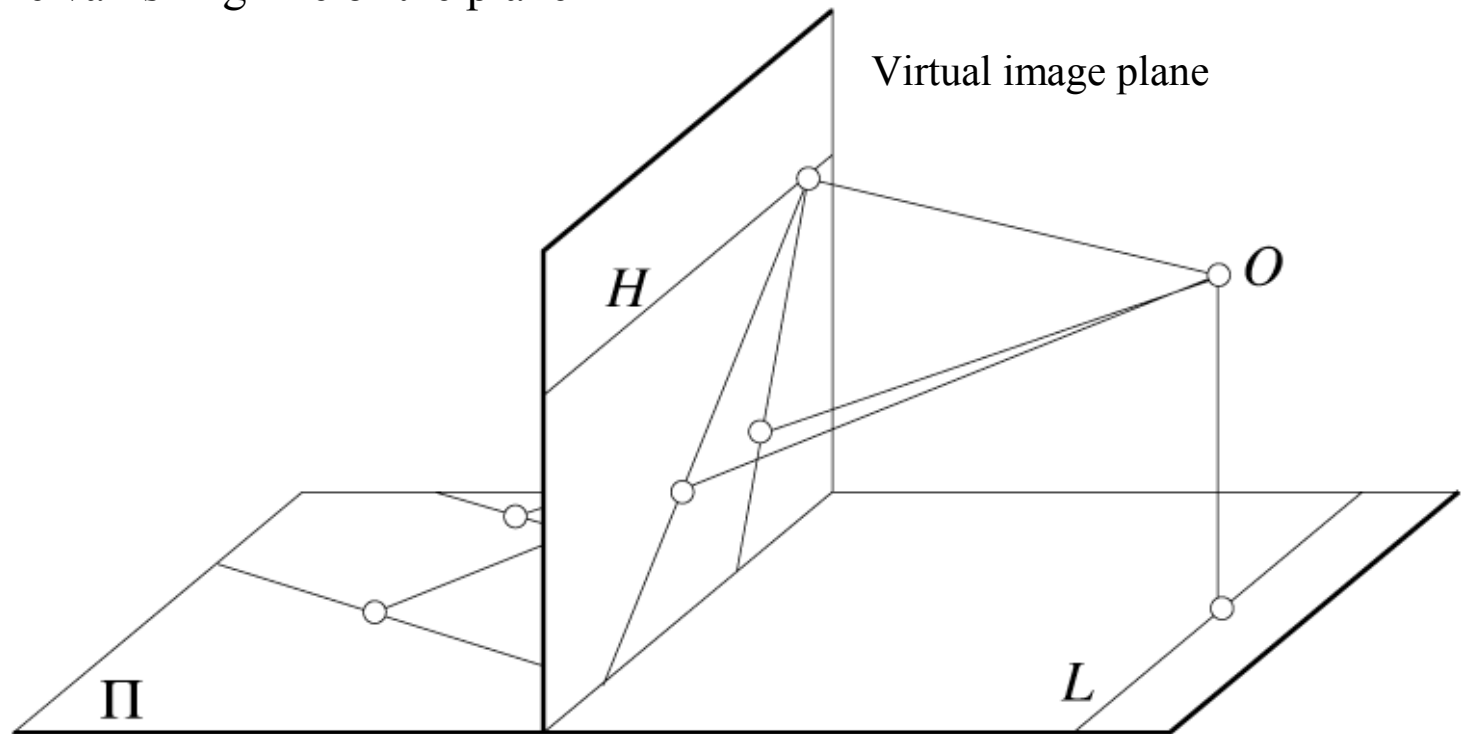


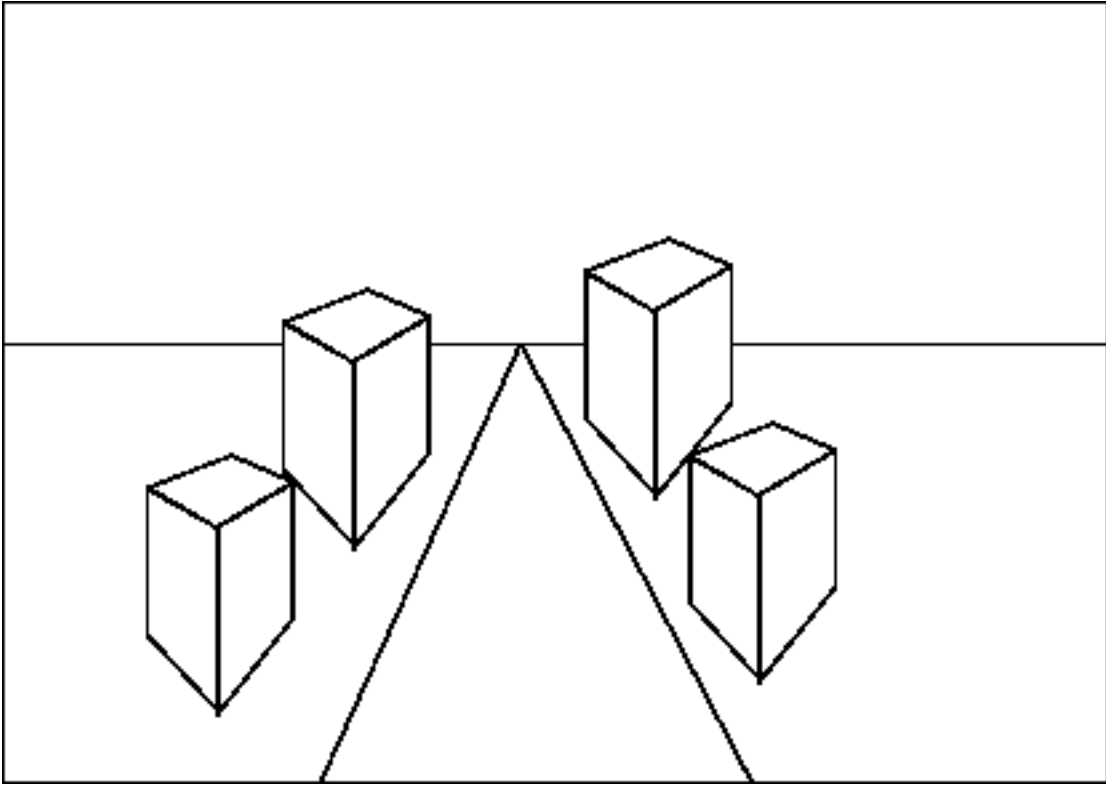
Adapted from Steven Seitz

Parallel lines meet

- Lines project to lines
- The projection of parallel lines meet at a single vanishing point
- Vanishing points of coplanar sets of lines are collinear, form the vanishing line of the plane (horizon)

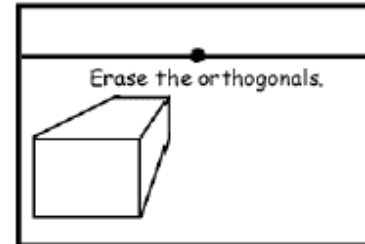
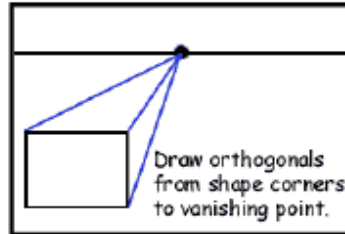
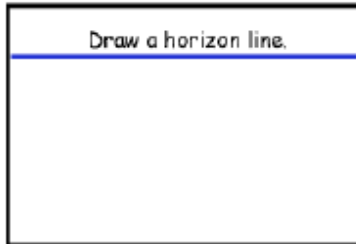
Common to draw film plane *in front* of the focal point. Moving the film plane merely scales the image.



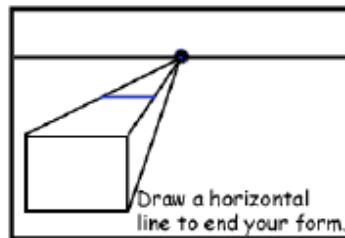
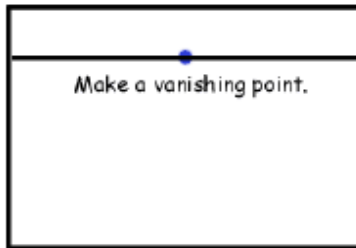


Adapted from David Forsyth, UC Berkeley

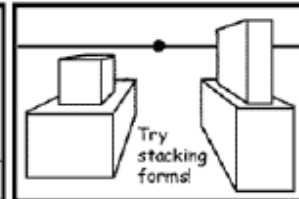
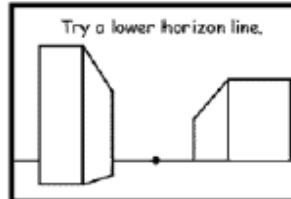
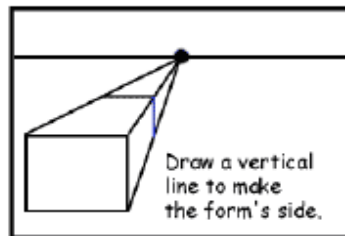
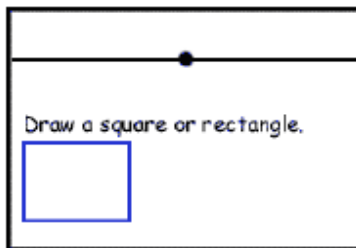
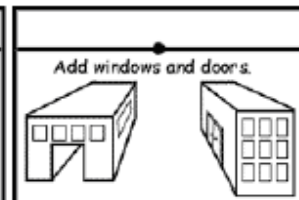
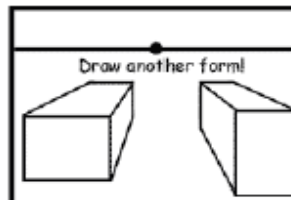
Drawing using vanishing points



Now you have a 3-D form in one-point perspective!



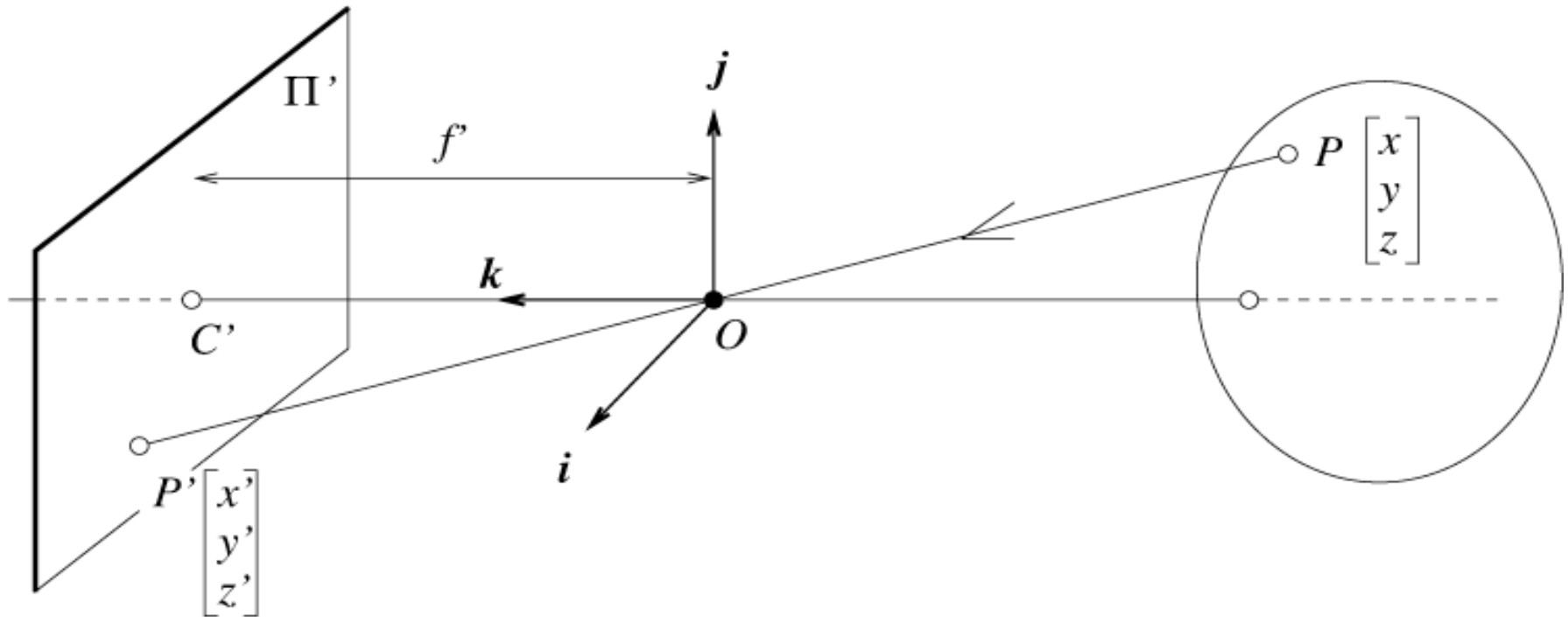
10. Add details and experiment!



http://www.sanford-artedventures.com/create/tech_1pt_perspective.html

Adapted from Trevor Darrell, MIT

Perspective Projection



Coordinate system (O, i, j, k) [O – origin, i, j, k – basis vectors]

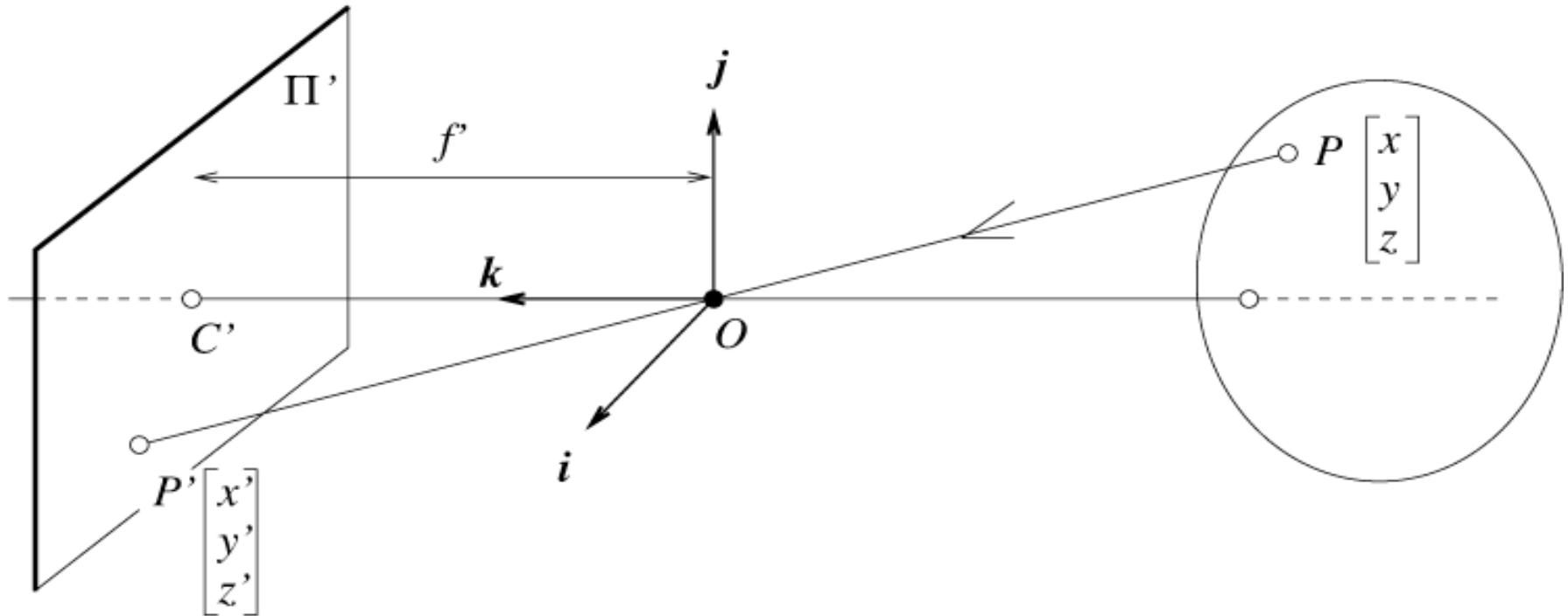
P : a scene point with coordinates (x, y, z) , P' : its image with coordinates (x', y', z')

Image plane Π' is located at a positive distance f' from the pinhole along the vector k

Line perpendicular to image plane and passing through the pinhole – optical axis

C' – image center

Perspective Projection



Since P' lies on image plane $\rightarrow z' = f'$

Since the points P, O, P' are collinear $OP' = \lambda OP$

$$x' = \lambda x$$

$$y' = \lambda y$$

$$f' = \lambda z$$

$$\lambda = x' / x = y' / y = f' / z$$

$$x' = f' \frac{x}{z}$$

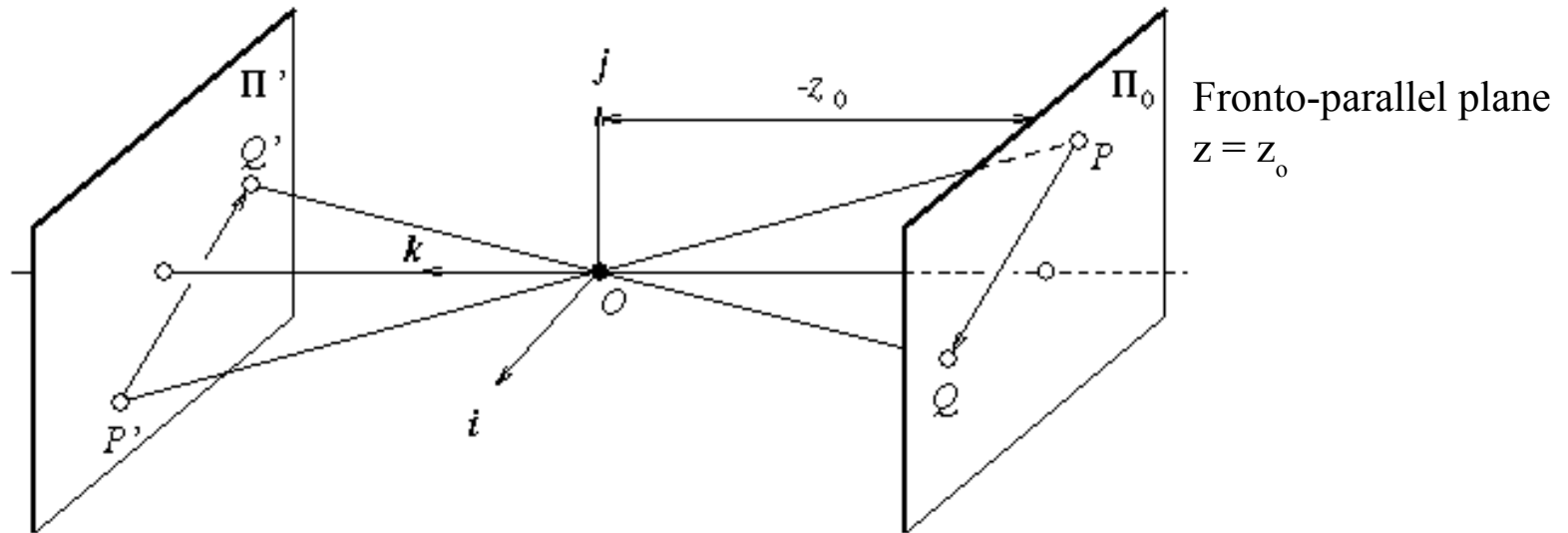
$$y' = f' \frac{y}{z}$$

The equation of projection

- Cartesian coordinates:
 - We have, by similar triangles, that
 $(x, y, z) \rightarrow (f' x/z, f' y/z, -f')$
 - Ignore the third coordinate, and get

$$(x, y, z) \rightarrow \left(f' \frac{x}{z}, f' \frac{y}{z} \right)$$

Special case : Affine Projection



For any point P in Π_0 we can write

$$\begin{aligned} x' &\approx -mx \\ y' &\approx -my \end{aligned} \quad m = -\frac{f'}{z_0}$$

If scene points are in a plane,
projections are simply
magnified by m

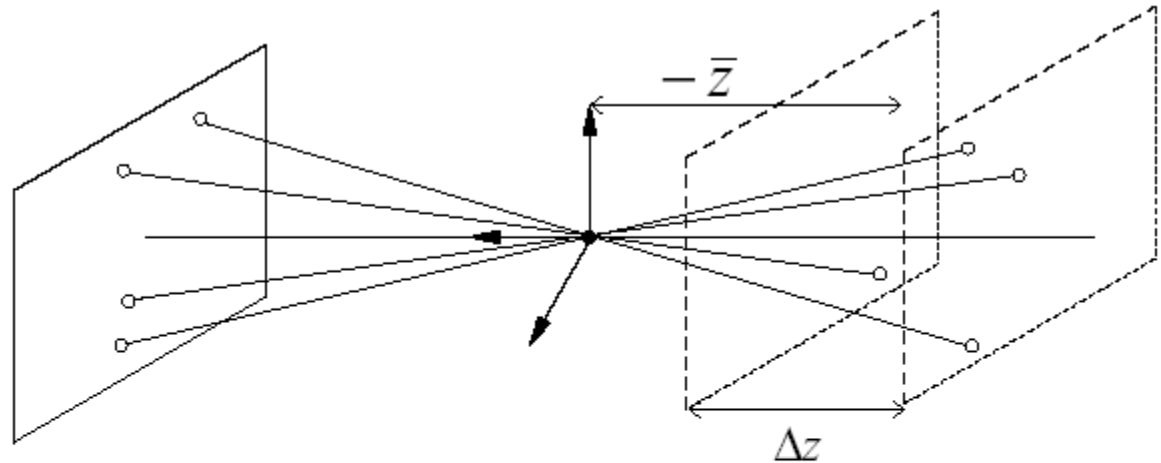
z_0 is negative, so magnification m is positive

$P'Q'$ & PQ are parallel and $|P'Q'| = m |PQ|$

Special case: Weak perspective

When the scene depth is small relative to the average distance from the camera, the magnification can be taken as constant
 -> weak perspective or scaled orthography

- perspective effects, but not over the scale of individual objects
- collect points into a group at about the same depth, then divide each point by the depth of its group
- Adv: easy
- Disadv: wrong



$$\text{If } \Delta z \ll -\bar{z}: \quad \begin{aligned} x' &\approx -mx \\ y' &\approx -my \end{aligned} \quad m = -\frac{f'}{\bar{z}}$$

Justified if scene depth is small relative to average distance from camera

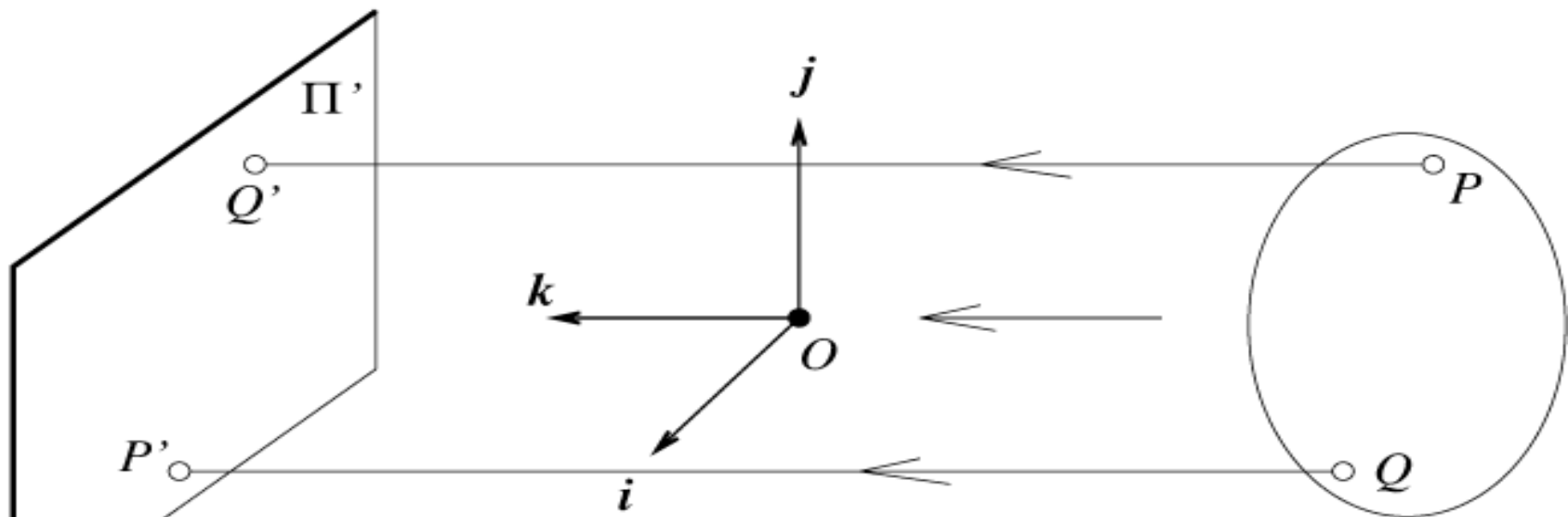
Extreme case: Orthographic projection

When it is known that the camera always remains at a roughly constant distance from the scene

Normalize coordinates $\rightarrow m = -1$

$$x' = x$$

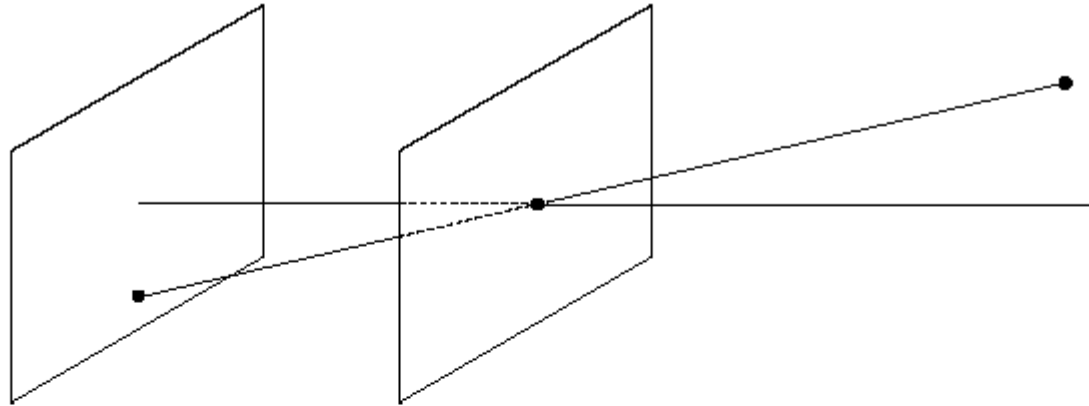
$$y' = y$$



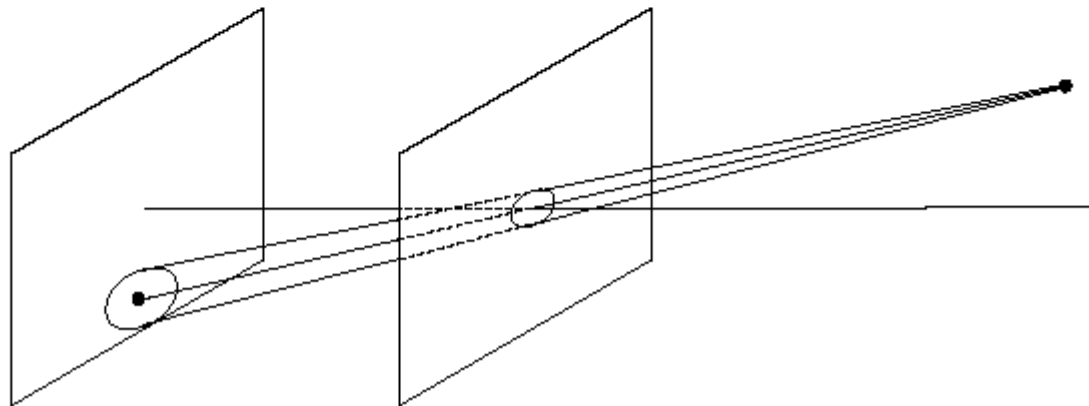
Justified if scene depth is small compared to distance from camera *and* camera remains at approximately constant distance

Limitations of the pinhole model

Ideal pinhole:
 Single scene point
 generates single image
 but:
 Diffraction
 Low light level

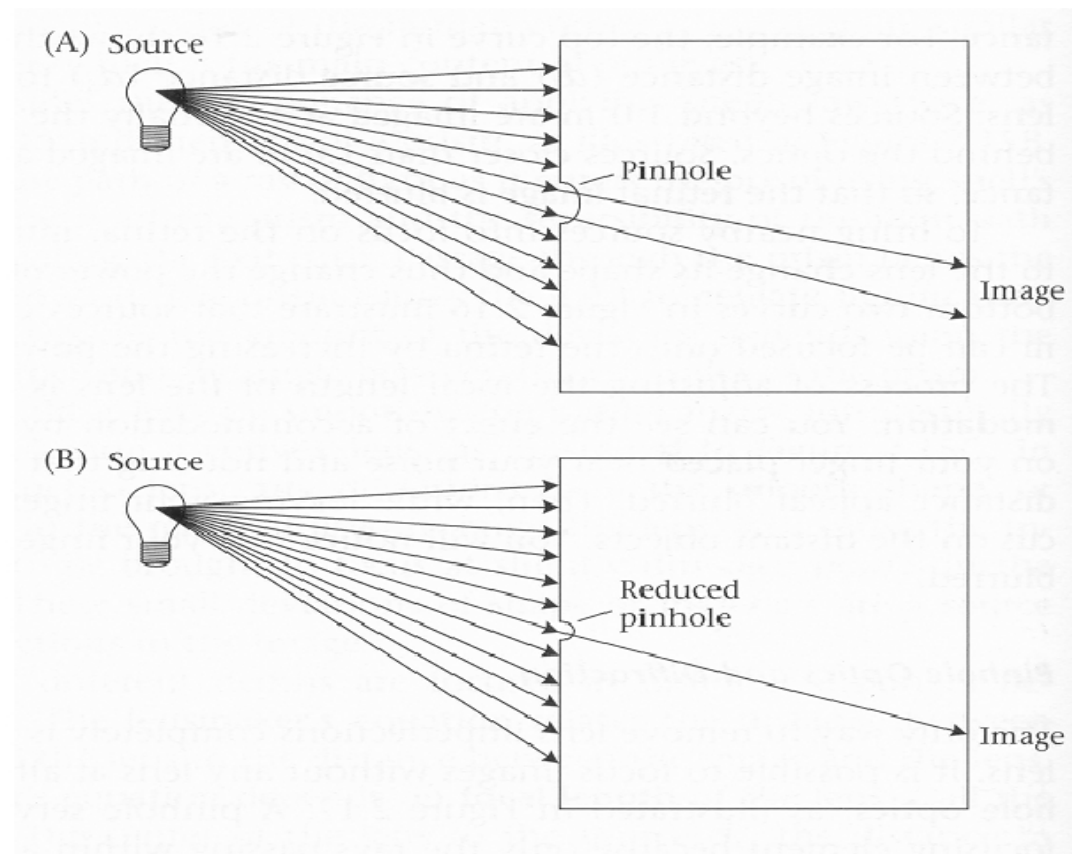


Finite-size pinhole:
 Single scene point
 generates extended
 image.
 Resulting image is
 blurry



Limitations of the pinhole model

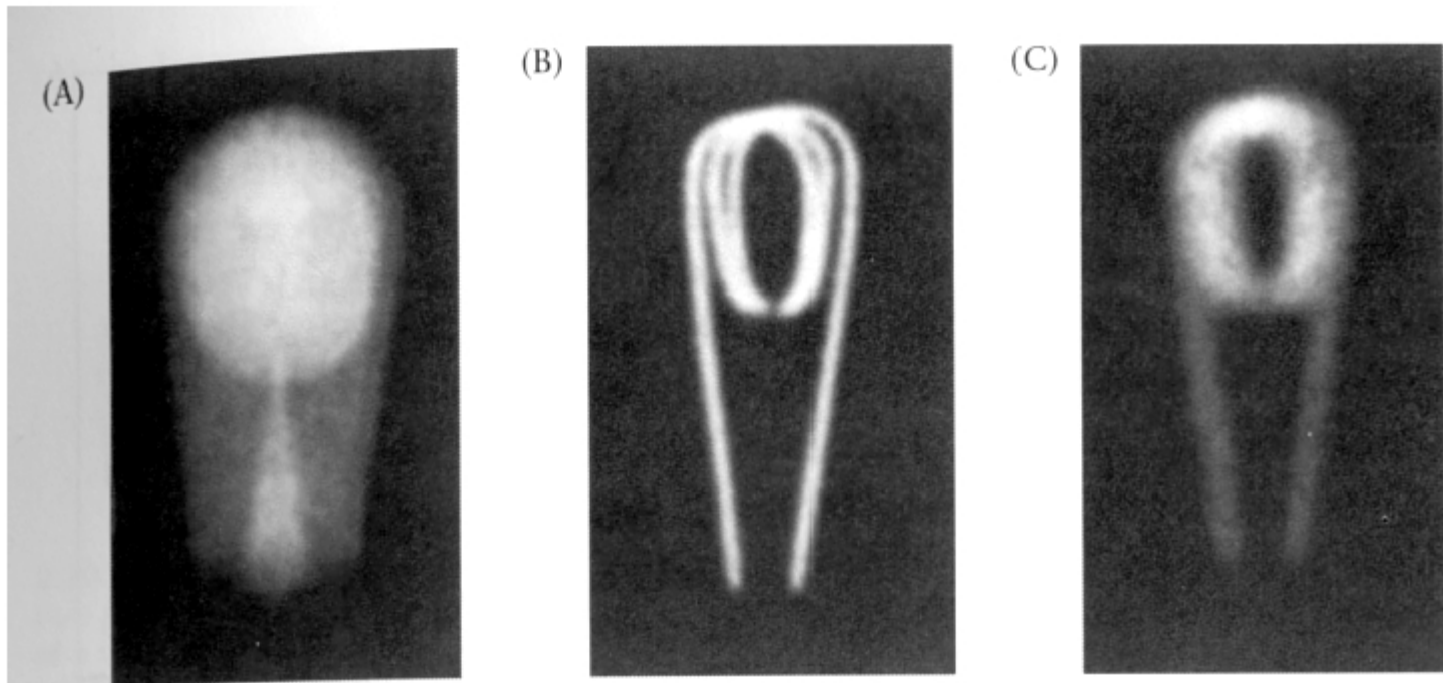
Larger the pinhole the wider the cone and brighter the image, but a large pinhole gives blurry image
Shrinking produces sharper images, but there may be diffraction



Wandell, Foundations of Vision, Sinauer, 1995

Adapted from Darrell and Freeman, MIT

Limitations of the pinhole model

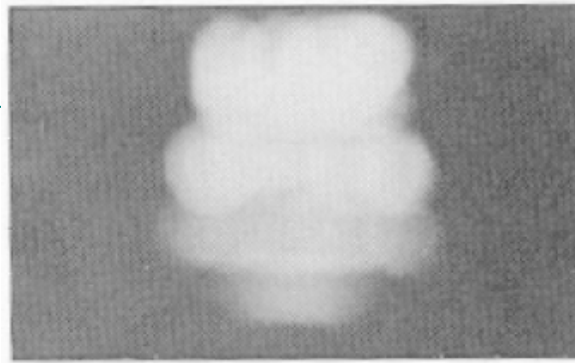


2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.

Wandell, Foundations of Vision, Sinauer, 1995

Adapted from Darrell and Freeman, MIT

Pinhole too big -
many directions are
averaged, blurring the
image



2 mm



1 mm

Pinhole too small-
diffraction effects blur
the image



0.6mm

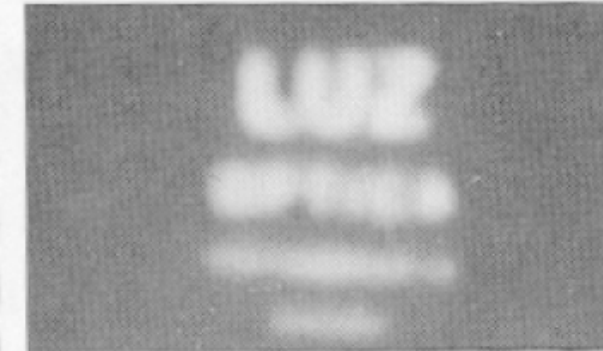


0.35 mm

Generally, pinhole
cameras are *dark*, because
a very small set of rays
from a particular point
hits the screen.



0.15 mm

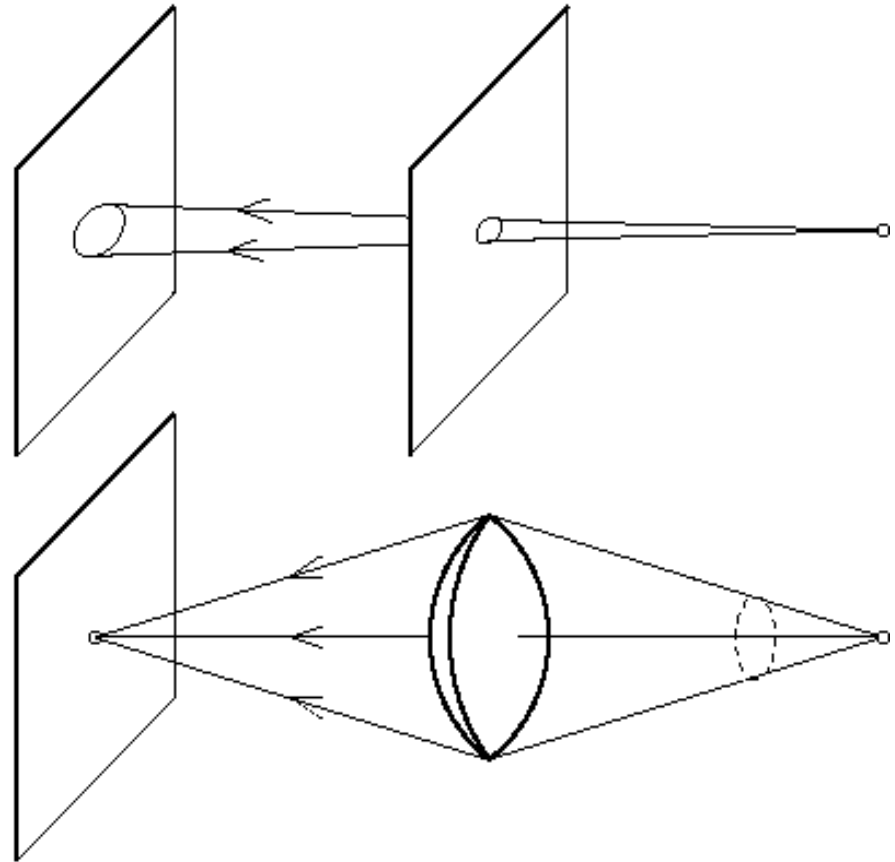


0.07 mm

The reason for lenses

To gather light since a single ray of light would otherwise reach each point in the image plane

To keep the picture sharp focus while gathering light from a large area



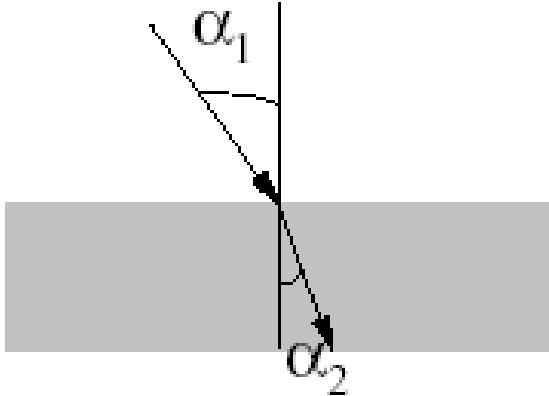
Laws of geometric optics

Light travels in straight lines in homogeneous media

when a ray is reflected from a surface, this ray, its reflection and the surface normal are coplanar, and the angles between the normal and the two rays are complementary

when a ray passes from one medium to another it is refracted (the direction changes)

Snell's law

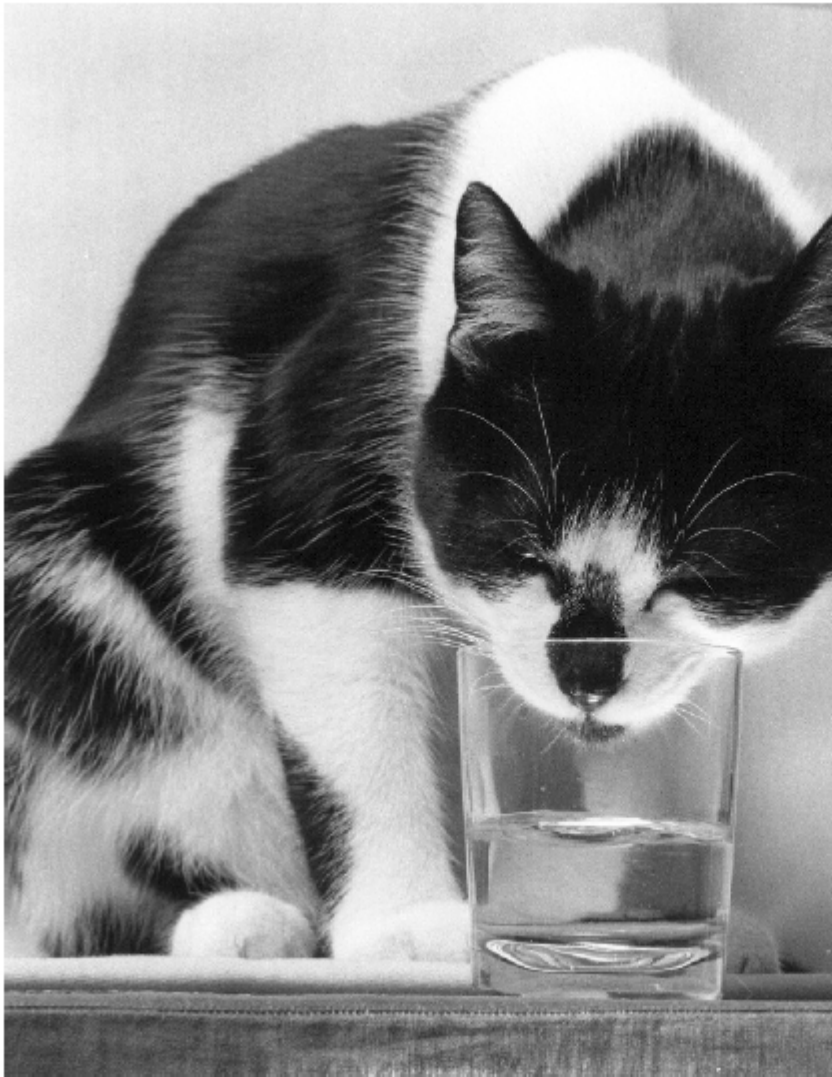


n_1, n_2 : indexes of refraction
 α_1, α_2 : angles between normal
and the rays

Snell's law of refraction:

$$n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2)$$

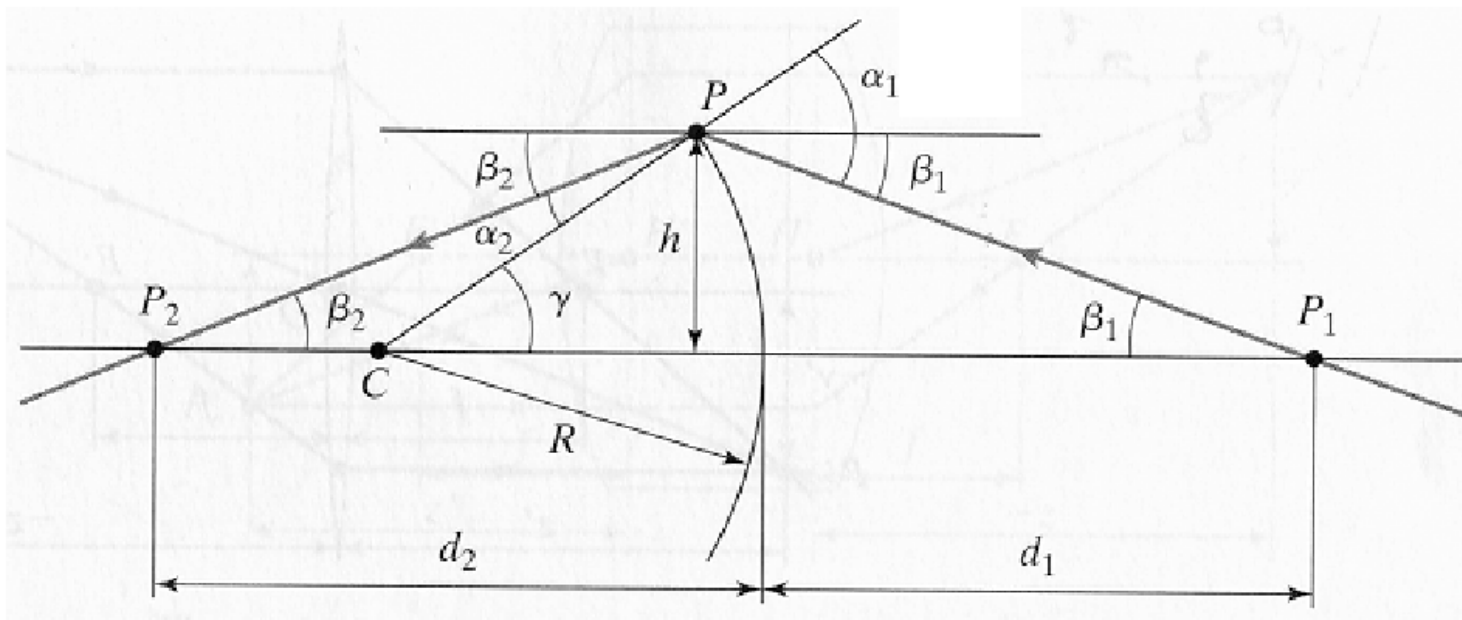
Water glass refraction



http://data.pg2k.hd.org/_exhibits/natural-science/cat-black-and-white-domestic-short-hair-DSH-with-nose-in-glass-of-water-on-bedside-table-tweaked-mono-1-AJHD.jpg

Adapted from Darrell & Freeman, MIT

Paraxial (first order) Geometric Optics



Consider an incident light ray passing through a point P_1 on the optical axis and refracted at the point P

h : distance between P and optical axis

R : radius of the circular interface

α_1 and α_2 : the angles between two rays and the chord joining C to P

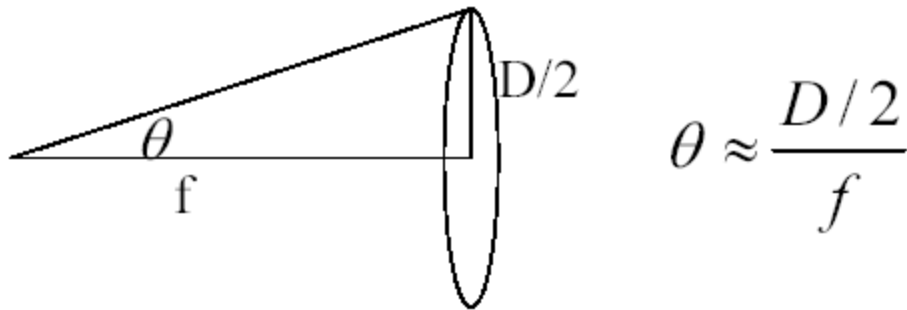
β_1 and β_2 : the angle between the optical axis and the line joining P_1 and P_2 to P

$$\gamma = \alpha_1 - \beta_1 = \alpha_2 + \beta_2$$

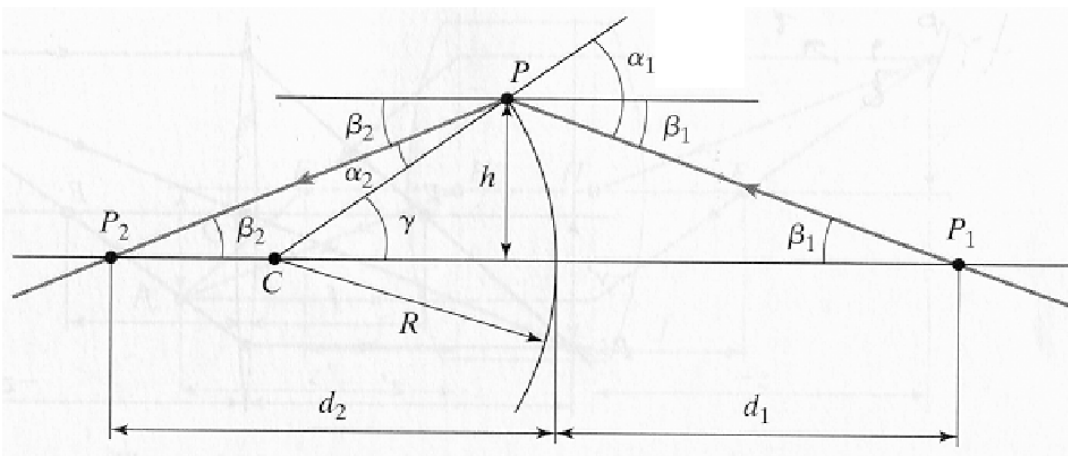
First Order Optics

Since all the angles are small they are equal to their sines and tangents

$$\sin(\theta) \approx \theta$$



Paraxial Geometric Optics



$$\alpha_1 = \gamma + \beta_1 \approx h \left(\frac{1}{R} + \frac{1}{d_1} \right)$$

$$\alpha_2 = \gamma - \beta_2 \approx h \left(\frac{1}{R} - \frac{1}{d_2} \right)$$

$$n_1 \alpha_1 \approx n_2 \alpha_2 \Leftrightarrow \frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R}$$

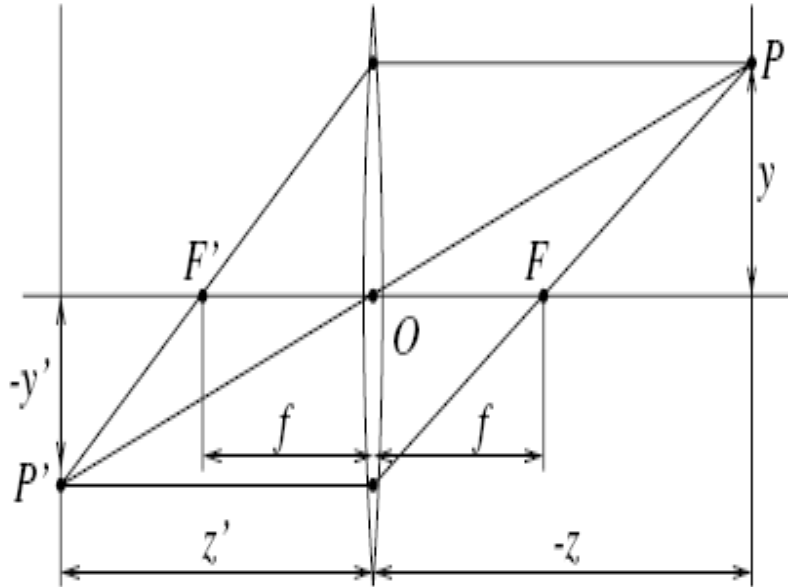
i.e. the relationship between d_1 and d_2 depends on R , n_1 and n_2 not to β_1 or β_2

Thin Lenses

Consider a lens with two spherical surfaces of radius R and index of refraction n

Assume that the lens is surrounded by vacuum ($n=1$)

it is thin : a ray entering the lens and refracted at its right boundary is immediately refracted again at the left boundary

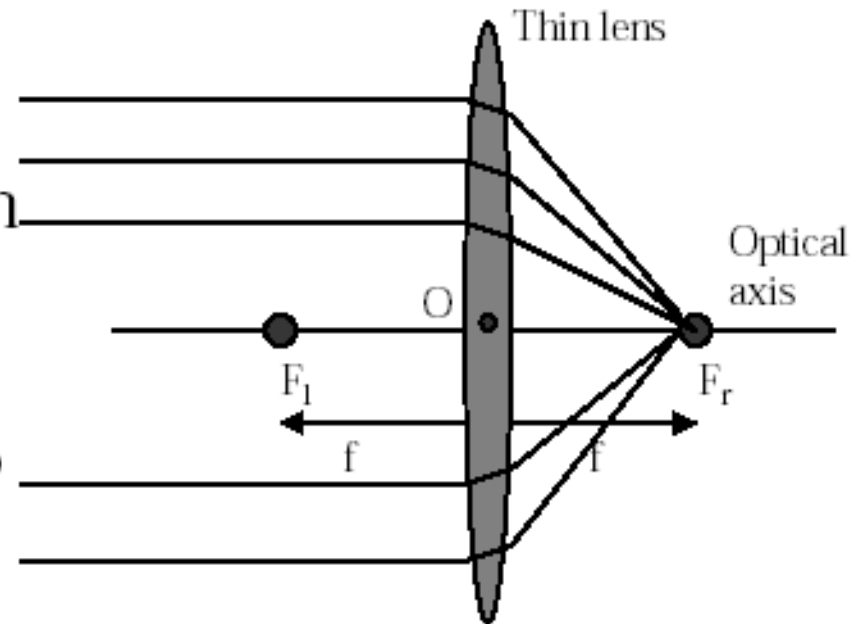


PO is not refracted
all other rays passing through P are focused by the thin lens on point P' with depth z'

Thin lenses

The shape of the lens is designed so that all rays parallel to the optical axis on one side are focused by the lens on the other side:

- Any ray entering the lens parallel to the optical axis on one side goes through the focus on the other side.
- Any ray entering the lens from the focus on one side, emerges parallel to the optical axis on the other side.



f : focal length

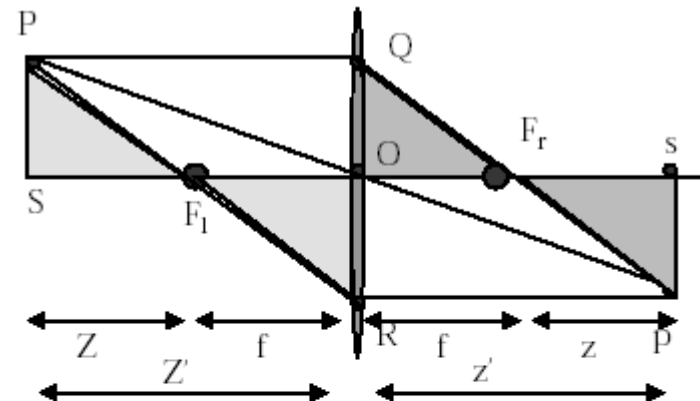
Thin Lenses

p can be determined by intersecting two known rays, PQ and PR.

- PQ is parallel to the optical axis, so it must be refracted to pass through F_r .
- PR passes through the left focus, so emerges parallel to the optical axis.

Note two pairs of similar triangles

- $PF_1S \leftrightarrow ROF_1$ and $psF_r \leftrightarrow QOF_r$



- $PS/OR = Zf$
- $QO/sp = f/z$
- But $PS = QO$ and $OR = sp$
- So, $Zf = f/z$, or $Zz = f^2$
- Let $Z' = Z + f$ and $z' = z + f$

$$\frac{1}{Z'} + \frac{1}{z'} = \frac{1}{f}$$

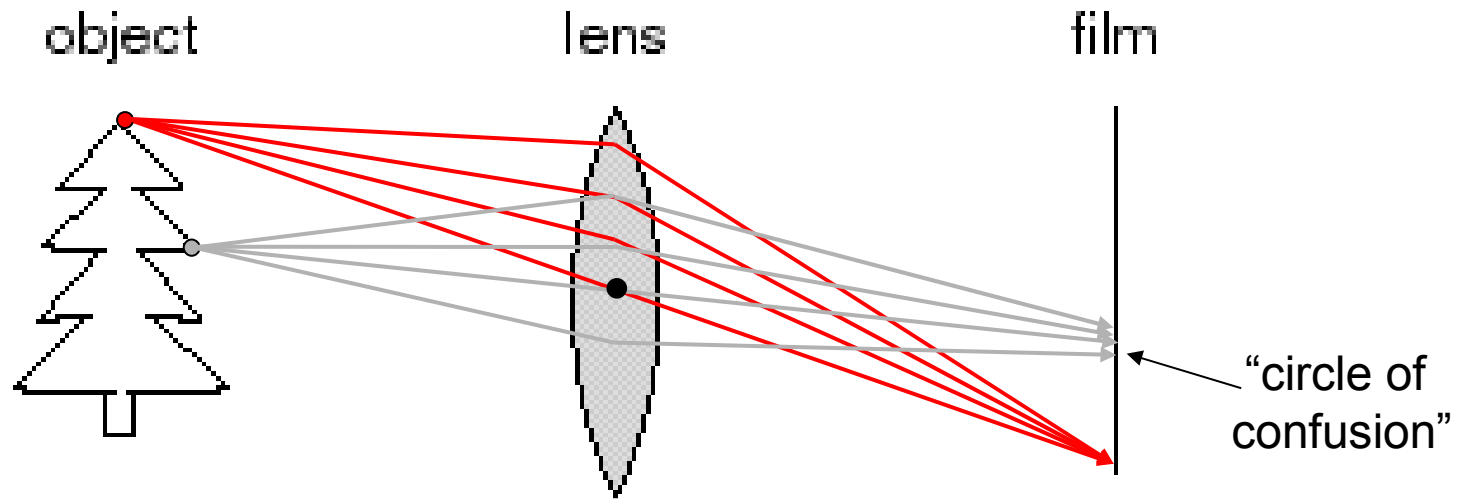
Thin lenses

Lens imperfections might cause these rays not to intersect at a point

Deviations in shape from ideal lens

Material imperfections that might cause the refractive index to vary within the lens

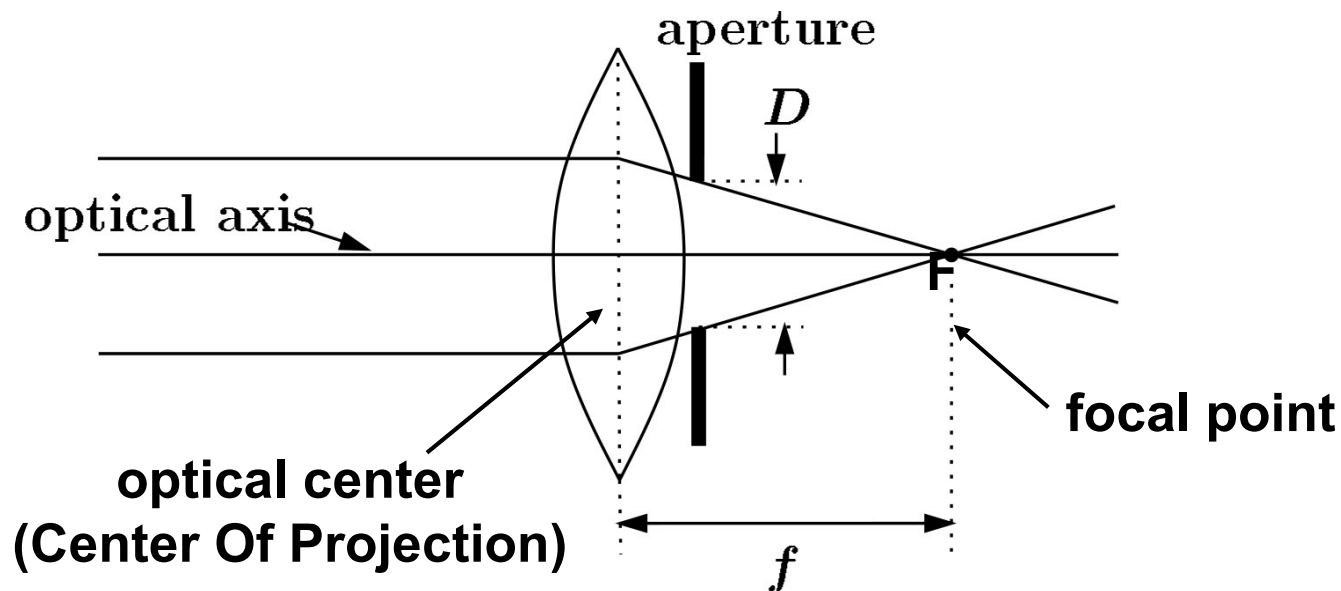
Thin lenses



- A lens focuses light onto the film
 - There is a specific distance at which objects are “in focus”
 - other points project to a “circle of confusion” in the image
- Changing the shape of the lens changes this distance

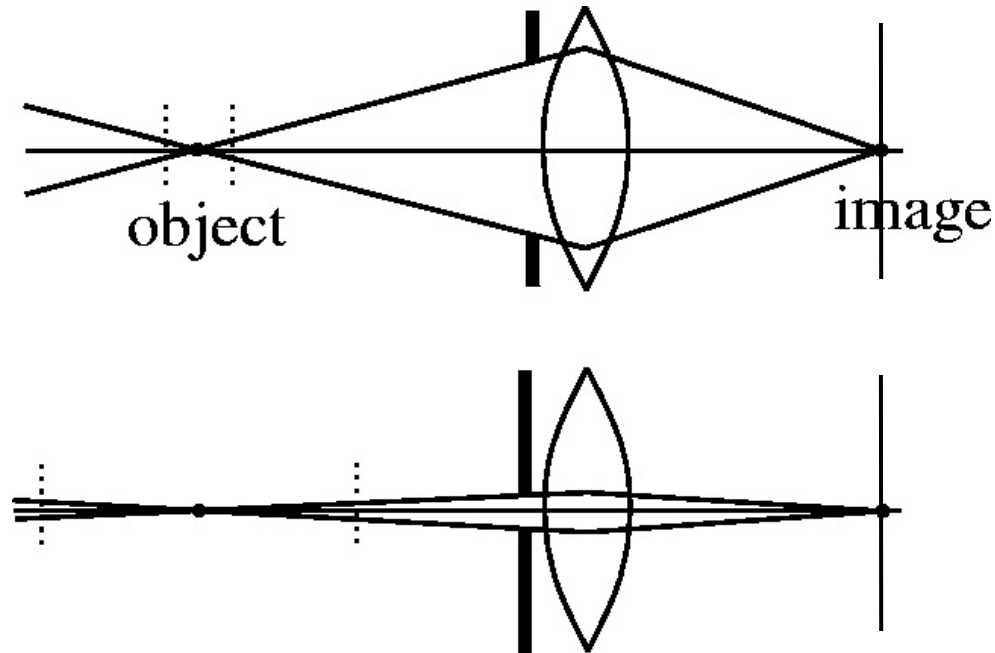
Adapted from Steven Seitz

Aperture



- A lens focuses parallel rays onto a single focal point
 - focal point at a distance f beyond the plane of the lens
 - f is a function of the shape and index of refraction of the lens
 - Aperture of diameter D restricts the range of rays
 - aperture may be on either side of the lens

Depth of Field



- Changing the aperture size affects depth of field
 - A smaller aperture increases the range in which the object is approximately in focus

Thick lens

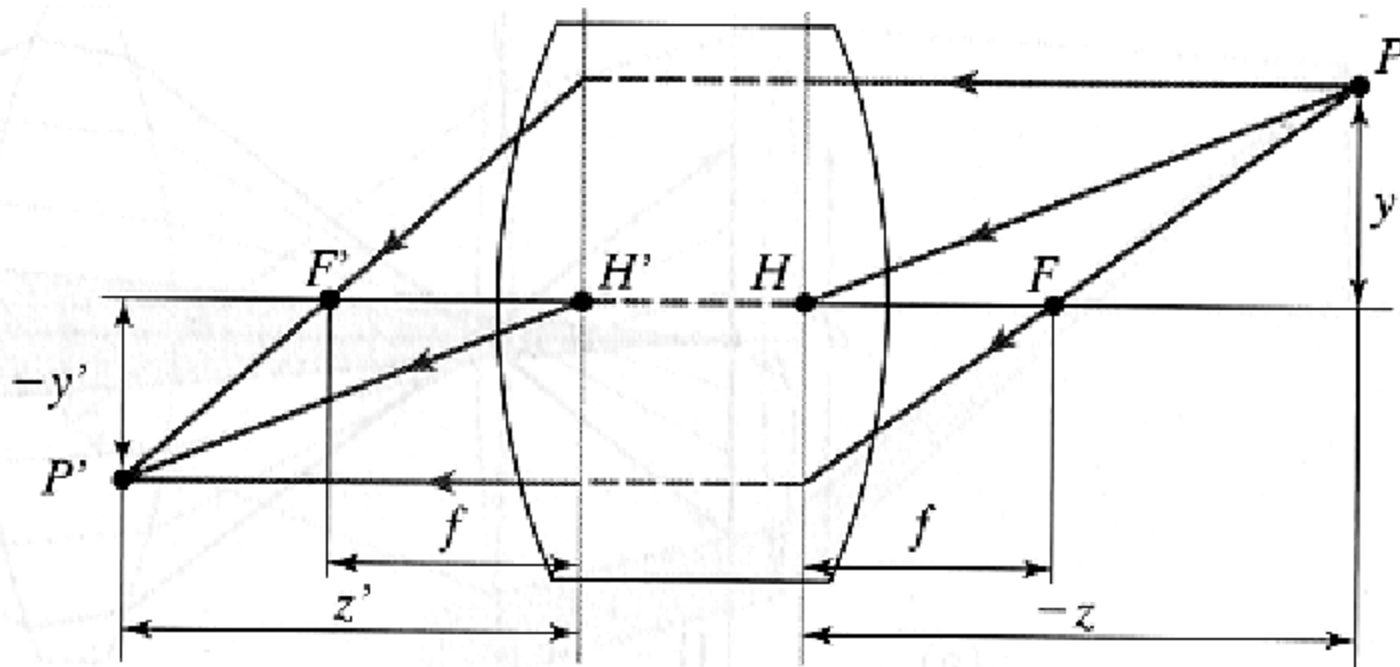
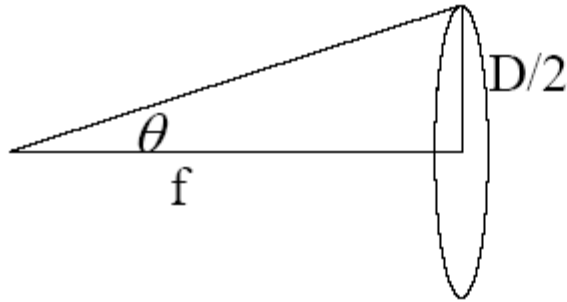


Figure 1.11 A simple thick lens with two spherical surfaces.

The only undeflected ray is along the optical axis

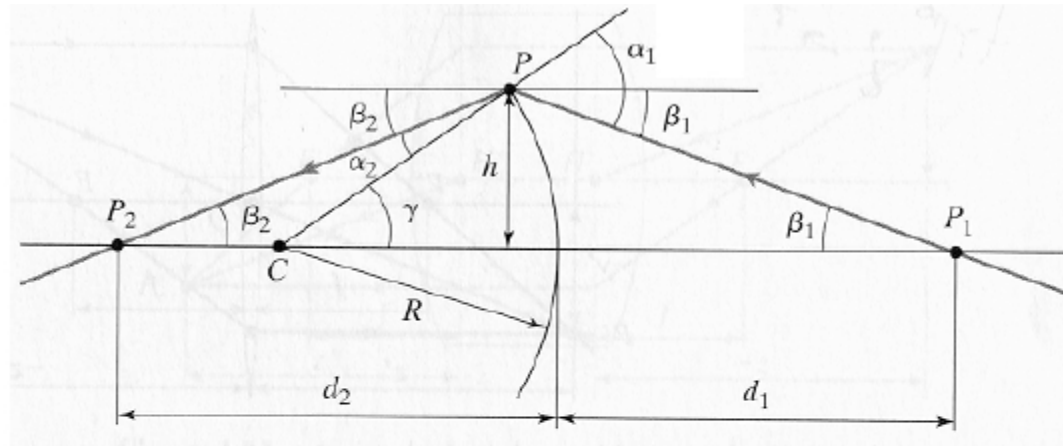
Third order optics

$$\sin(\theta) \approx \theta - \frac{\theta^3}{6}$$



$$\theta \approx \frac{D/2}{f} - \frac{\left(\frac{D/2}{f}\right)^3}{6}$$

Third order optics

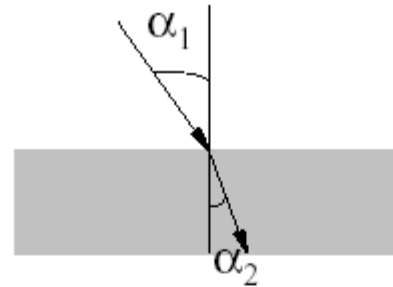


$$\frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R} + h^2 \left[\frac{n_1}{2d_1} \left(\frac{1}{R} + \frac{1}{d_1} \right)^2 + \frac{n_2}{2d_2} \left(\frac{1}{R} - \frac{1}{d_2} \right)^2 \right]$$

Aberrations

Snell's law of refraction:

$$n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2)$$



First-order optics: appropriate for ideal model of thin lens

$$n_1 \alpha_1 \approx n_2 \alpha_2$$

Higher order optics: necessary for real lenses

Taylor expansion

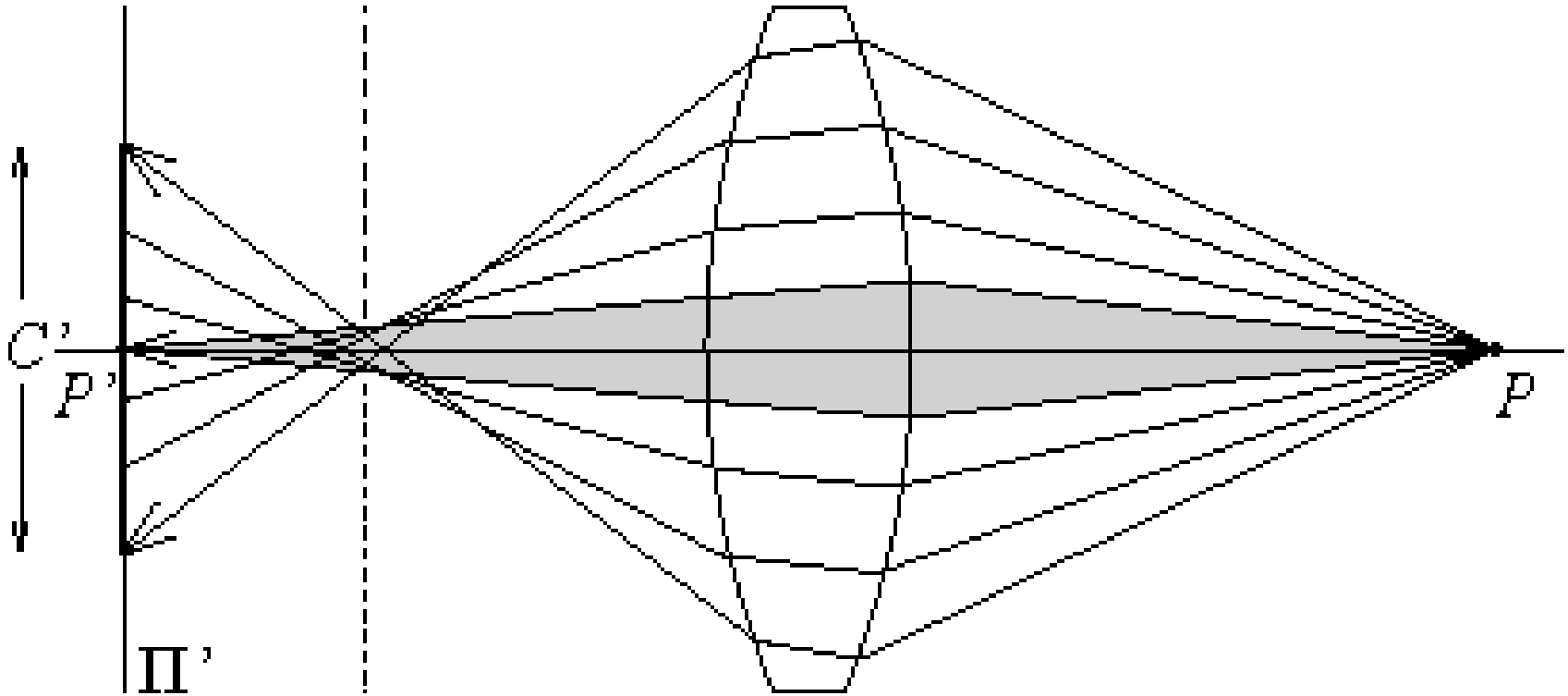
$$\sin \alpha \approx \alpha - \frac{\alpha^3}{6} + \dots$$

Aberrations:

Blurring: e.g., spherical aberrations

Geometric distortion

Spherical aberration



Circle of least confusion

Chromatic aberration

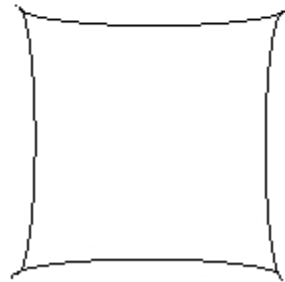
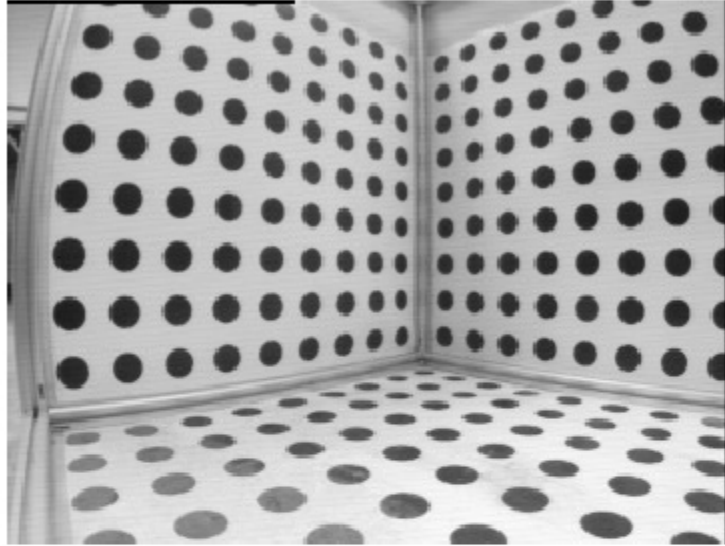
(great for prisms, bad for lenses)



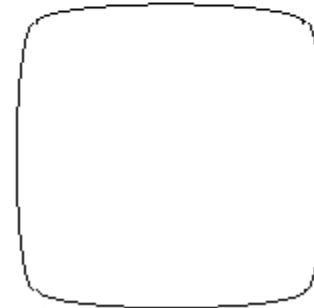
- Chromatic aberration
 - Light at different wavelengths follows different paths; hence, some wavelengths are defocussed

Adapted from Trevor Darrell, MIT

Geometric Distortion

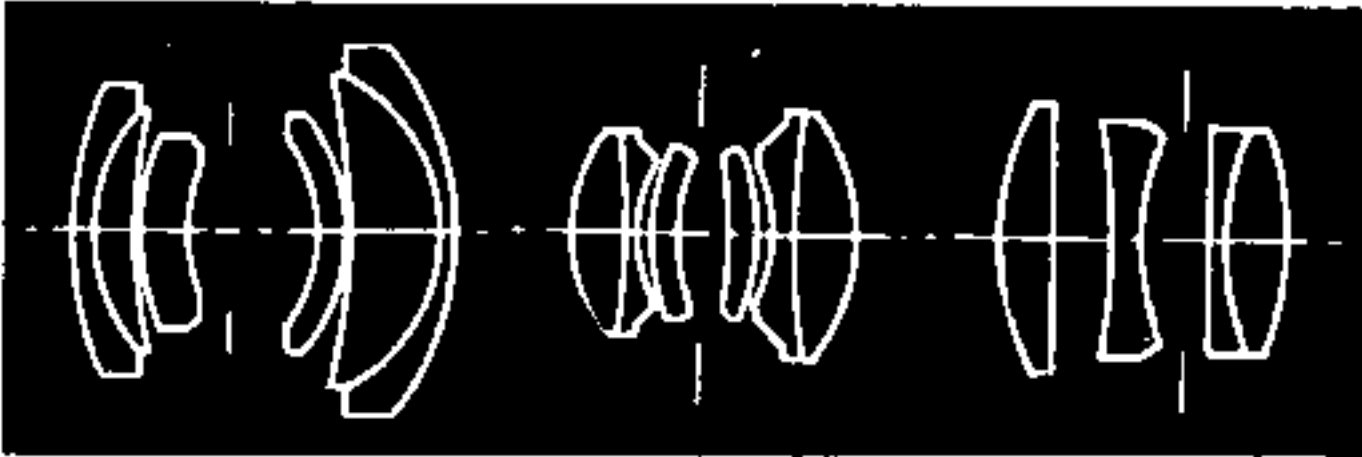


pincushion



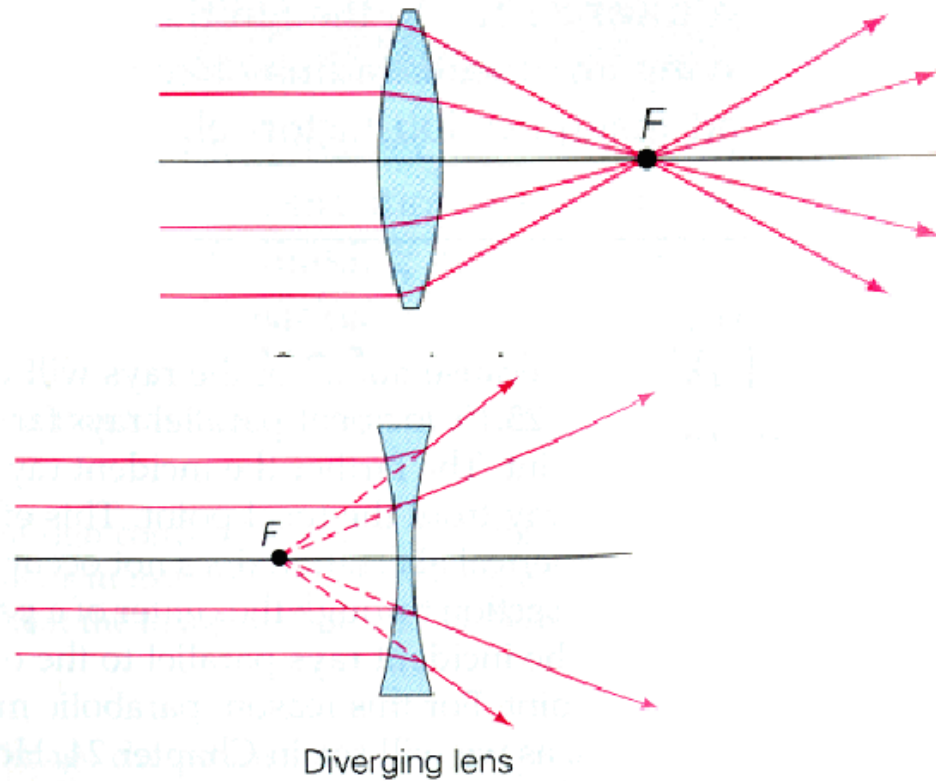
barrel

Lens systems



Adapted from David Forsyth, UC Berkeley

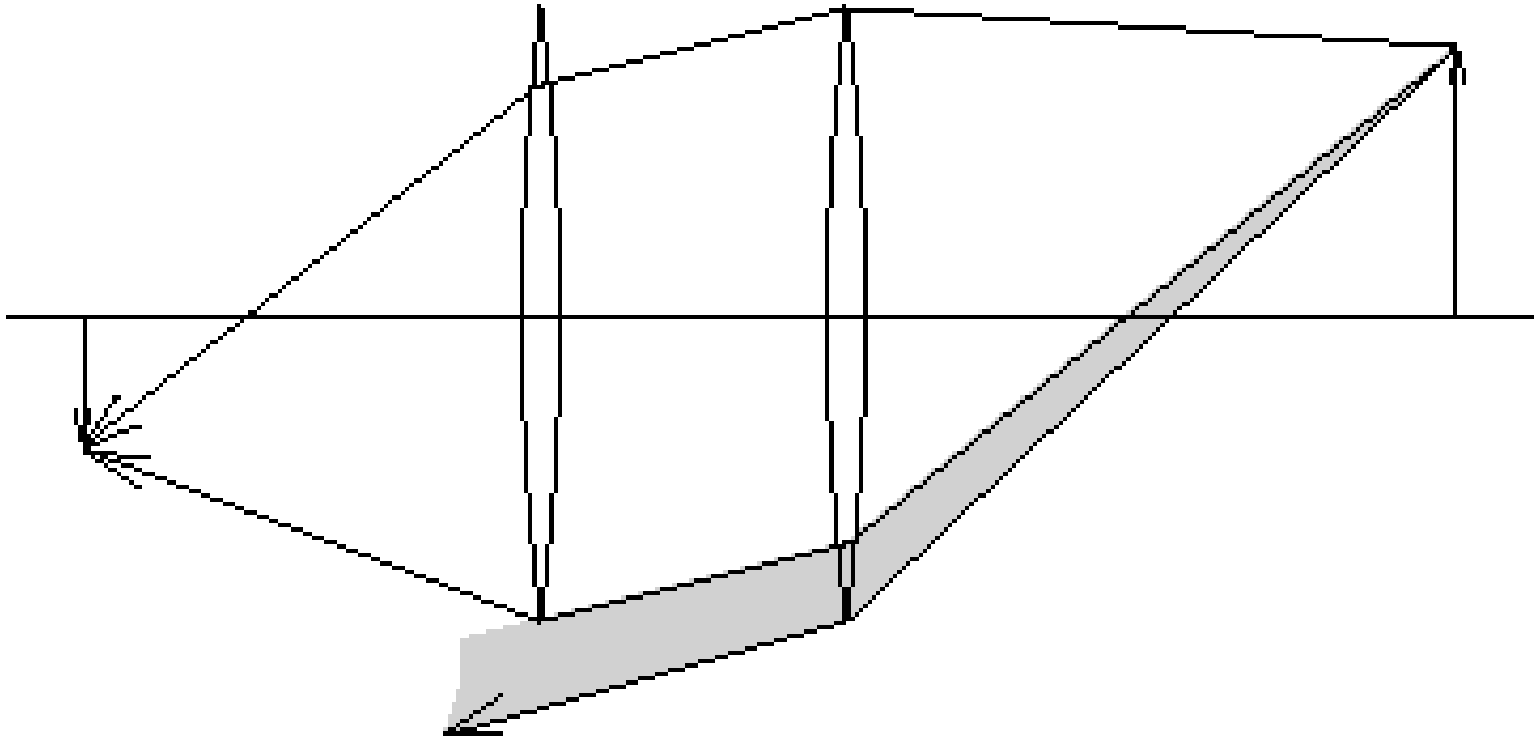
Convex and concave lenses



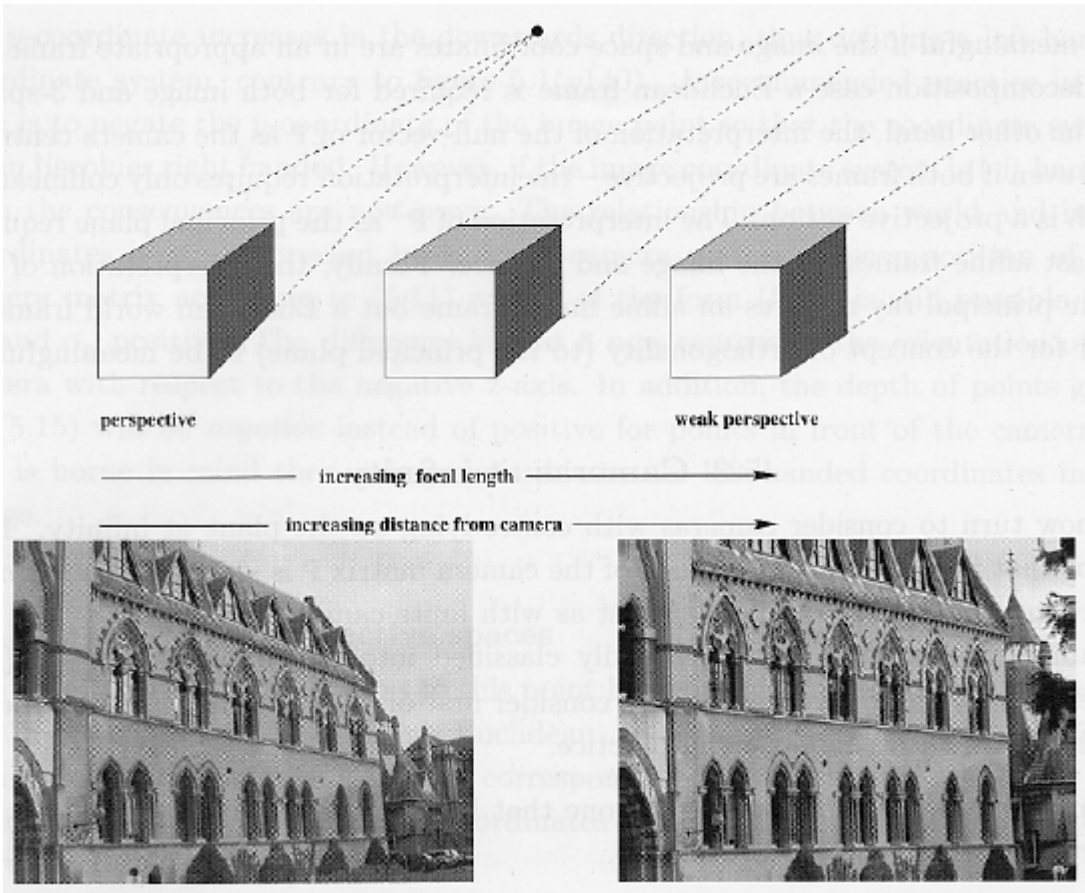
<http://www.physics.uiowa.edu/~umallik/adventure/light/lenses.gif>

Adapted from Freeman & Darrell, MIT

Vignetting



Causes the brightness to drop in the image periphery



Adapted from Martial Hebert, CMU



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CCD cameras

- A digital camera replaces film with a sensor array
 - Each cell in the array is a **Charge Coupled Device**
 - light-sensitive diode that converts photons to electrons

A CCD sensor uses a rectangular grid of electron-collection sites laid over a thin silicon wafer to record a measure of the amount of light energy reaching each of them

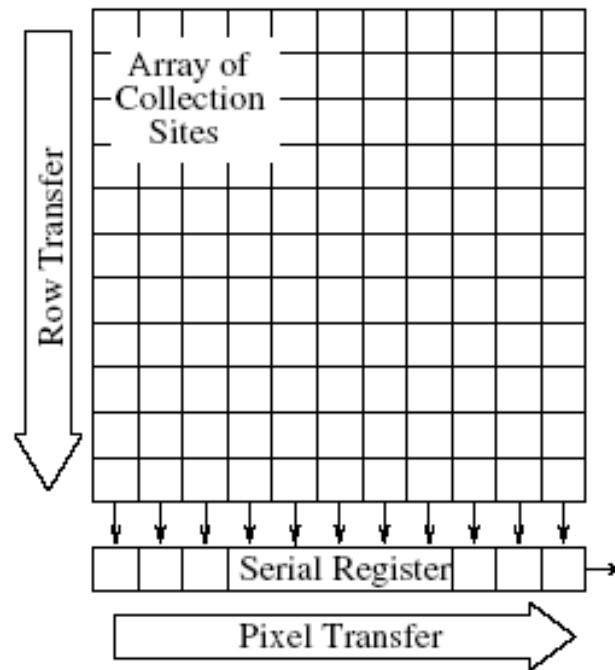


Figure 1.24. A CCD Device.